Note: From this point forward I will write only *selected solutions*. I will provide solutions to every problem that some one had trouble with in which case the mistake was not obvious (e.g. a typo) or something I was not able to explain on that student's problem sheet. Feel free to contact me with additional questions.

2.c,j (pp 24-25)

c. "Turn off the music or I'll scream" is a proposition. This statement is equivalent to $P \vee Q$ where P is the statement "you turn off the music" and Q is the statement "I will scream." This threat is either true or false, so it is a proposition.

j. "Goldbach's Conjecture is true" is a proposition. True, we do not know whether or not it is a true proposition, but if it is proven, it will be proven true or false so in the meanwhile this is a proposition which is simply unverifiable.

3 (pp 139-140)

Sets a, b, c, f, and g are all equal. For $a = b$ the axiom of extensionality says that set membership is all that can determine the identity of a set. Thus these sets are also equal to c because writing the same element twice does not change the set's membership. With simple arithmetic, it is easy to see that $f = \{1, 2, 3\} = a$ as well. Finally the symbol $\sqrt{x}$ where $x \in \mathbb{R}$ is greater than zero is generally interpreted to mean the positive square root of this element. So g also equals a.

The set d includes non-integers so it is distinct from all of the others. The set e includes the negative integers $-1, -2, -3$ and thus is also distinct from the others.

6.c

The set $\{x \in \mathbb{R} \mid x \geq 5 \text{ or } -x < -2\} = \{x \in \mathbb{R} \mid x \geq 5 \text{ or } x > 2\} = [5, \infty) \cup (2, \infty) = (2, \infty)$.

9

i and ii. The statement $\exists x, y(x^2 - y^2 = 2)$ is false in $\mathbb{Z}$ and thus also $\mathbb{N}$. To see this note that $x^2 - y^2 = (x - y)(x + y)$. If $x, y \in \mathbb{Z}$ then so are $x - y$ and $x + y$. So if there exist $x, y \in \mathbb{Z}$ such that $x^2 - y^2 = 2$ then $2 = (x - y)(x + y)$. However, 2 factors uniquely in the integers. Thus either $x - y = \pm 1$ and $x + y = \pm 2$ or $x - y = \pm 2$ and $x + y = \pm 1$. (Note in each pair of equations the $\pm$ is intended to be either $+$ in both paired equations or $-$ in both paired equations so that the product is positive.) In the first case, adding the equations shows that $2x = \pm 3$ so $x \notin \mathbb{Z}$. In the second case, $2x = \pm 3$ as well and is again not in $\mathbb{Z}$. Thus no solution may exist.

iii and iv. The above computation suggests a way to find a solution with $x, y \in \mathbb{Q}$ (and thus also a solution in $\mathbb{R} \supset \mathbb{Q}$). The factorization $2 = (x - y)(x + y)$ holds in $\mathbb{Q}$ as well. Here it is not necessary that the terms $x - y$ and $x + y$ be integers, but it is certainly possible. If we set $x + y = 2$ and $x - y = 1$ then as before $2x = 3$ and $x = \frac{3}{2}$. So we see $y = \frac{1}{2}$ and sure enough

$$(\frac{3}{2})^2 - (\frac{1}{2})^2 = \frac{8}{4} = 2$$

is a solution in $\mathbb{Q}$. Clearly this solution exists in $\mathbb{R}$ as well, though there are more obvious ones.