Math S–101. Worksheet 7.
Topological Spaces

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Summer 2006

The Closure Axioms

A *closure operator* $\mathbf{K}$ on a set X is a rule that associates to each subset A of X, a subset of X denoted by $\mathbf{K}(A)$ or simply $\mathbf{K}\,A$, such that following axioms hold for all subsets A and B of X.

1. $A \subset \mathbf{K}(A)$
2. $\mathbf{K}(A \cup B) = \mathbf{K}(A) \cup \mathbf{K}(B)$
3. $\mathbf{K}(\mathbf{K}(A)) = \mathbf{K}(A)$
4. $\mathbf{K}(\emptyset) = \emptyset$

A *topological spaces* is a pair $(X, \mathbf{K})$, where X is a set and $\mathbf{K}$ is a closure operator on X.

Problems

1. Let X be a topological space with closure operator $\mathbf{K}$. If $\mathbf{K}(A) \subset A$, then $\mathbf{K}(A) = A$.
2. If X is a topological space with closure operator $\mathbf{K}$, then $\mathbf{K}(X) = X$.
3. Let X be a topological space with closure operator $\mathbf{K}$. If $A \subset B$, then $\mathbf{K}(A) \subset \mathbf{K}(B)$. *Hint*: Use the fact that $A \subset B$ if and only if $A \cup B = B$.

4. Let X be a topological space with closure operator $\mathbf{K}$. Then $\mathbf{K}(A \cap B) \subset \mathbf{K}(A) \cap \mathbf{K}(B)$.