Mathematics 113 Fall 2002

HOMEWORK ASSIGNMENT # 2
DUE, Thursday, October 3

Collaboration: On the homework sets, collaboration is not only allowed but encouraged. However, you must write up and understand your own individual homework solutions, and you may not share written solutions. If you learn how to solve a problem by talking to a classmate, CA, or looking it up in a book, you should cite the source in your homework write-up, just as you would reference your sources in a literature or history class.
Show all your working, and write up your solutions as neatly as possible.

1. Define $\partial/\partial z$ and $\partial/\partial \overline{z}$ by setting

$$\partial f/\partial z = \frac{1}{2}\left(\partial f/\partial x - i \cdot \partial f/\partial y\right)$$

   and

$$\partial f/\partial \overline{z} = \frac{1}{2}\left(\partial f/\partial x + i \cdot \partial f/\partial y\right)$$

   Show that the Cauchy–Riemann conditions are equivalent to $\partial f/\partial \overline{z} = 0$. Moreover, show that if $f(z)$ is holomorphic then $f'(z) = \partial f/\partial z$.

2. Let $f(x+iy)$ be a polynomial (with complex coefficients) in x and y. Show that $f(x+iy)$ is holomorphic if and only if it can be expressed as a polynomial in the single variable $z = (x+iy)$. (Hint: first write f as a polynomial in z and $\overline{z} = x - iy$.)

3. Prove or disprove:

$$\lim_{z\to 0} z \cdot \sin\left(\frac{1}{z}\right) = 0.$$

4. Let $f(z)$ and $g(z)$ be holomorphic functions. Prove that the composition $f(g(z))$ is also holomorphic, and that its derivative is $f'(g(z))g'(z)$.

5. Show that the inverse tangent can be written as

$$\tan^{-1} z = \frac{1}{2i}\log\left(\frac{1+iz}{1-iz}\right).$$

   Find a branch cut that makes this function holomorphic.