

Math 113 Problem Set 11

Due Wednesday, December 10, 2003

1. Bak and Newman, Chapter 14, Problem 5.
2. Bak and Newman, Chapter 14, Problem 9.
3. In class, we saw how to construct a Riemann mapping from the upper half plane to a polygon with specified angles at vertices. (This is the Schwarz-Christoffel formula.) However, to get a specified shape of the final polygon, we need to pick appropriate points on the boundary of the upper half plane. In this problem, we will make these choices more explicit to construct a map from the upper half plane to the square.

(a) Show that the function

$$f(z) = \int_0^z \frac{dw}{\sqrt{w(w^2 - 1)}}$$

maps the upper half plane to a rectangle R .

(b) Consider the map

$$g(z) = f\left(\frac{z+i}{iz+1}\right)$$

Show that $g(z)$ maps the unit disk to R . What points on the boundary of the unit disk get mapped to the vertices of R ? Use symmetry to show that R is actually a square.

(c) Find an explicit map from the upper half plane to a hexagon.