Mathematics 116
Convexity and Optimization with Applications

Assignment V Due in class on November 3.

Announcements In addition to office hours and sections, help is also available on Sundays at 8pm at the Math Question Center in Loker Commons. Note: The midterm will be in class, but not on November 10.

Reading Chapters 3 and 4 of Luenberger. Look at Problem 3.12 for section.

Exercises Do four from among Luenberger §3.13 #7 and 8, 16, 17, 20, 21.

Writing Please include your answers with your problem set.

1. Consider the step function *x(t)* that equals 1 for $-\pi \le t < 0$ and equals 0 for $0 \le t \le \pi$. Show that its Fourier series is $\frac{1}{2}+\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{((-1)^n-1)}{n}\sin(nt)$. Discuss what this means. I.e., in what sense does this converge? In $L_2[-\pi,\pi]$? Pointwise? Uniformly? How do you know? What is the significance of the constant term? Why are the coefficients of the cosine terms all zero?

2. In Example 1 of §3.8, use Gram-Schmidt to work out the first three Legendre polynomials and show that they agree with the formula given. Can you think of or look up uses for this sequence?

Discussion For discussion in section. You are also encouraged to post what you find to the discussion section of our website, along with any questions or observations you might also have.

1. For a sequence $\{x_n\}$ in $L_2[-\pi,\pi]$, we have seen three different ways we could say it converges: pointwise, uniformly, and in the L_2 norm (sometimes called convergence in mean). Give arguments and examples that illustrate the relations between these notions. E.g., show that uniform convergence implies pointwise convergence, but give an example to show the converse is not true, then show that pointwise convergence does not imply L_2 convergence, etc.

2. Ok, let's be more explicit about why 1 + 1/4 + 1/9 + 1/16 + …. converges to $\pi^2/6$. This is not what Euler did, but try computing the Fourier series of x(t) = t in $L_2[-\pi,\pi]$ then use Parseval's Identity (Problem 16). Can you show that $\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{90}$? Others?