# Mathematics 116
# Convexity and Optimization with Applications

Review Suggestions

Details: The midterm will be in class on December 3.
No notes or calculators are allowed.

Advice: Be able to state carefully major theorems and definitions.

Go over the homework and understand it, but do not be too concerned about technicalities and detailed arguments.

Be prepared to answer some true/false questions, to explain your answers, and to give examples.

Simple calculations with numbers may be useful, too.

All of Chapters 1 through 5 of Luenberger are fair game, except Chapter 4, where you are responsible only for the bits gone over in class and in the homework.

Think:

1. Say if the following statements are always true, true for finite dimensions, or not generally true. (As always, assume that our scalars are the real or complex numbers.)

a. Subspaces are complete.
b. Given a vector space, there exists a basis.
c. Linear functions are continuous.
d. The unit ball is compact.
e. The unit ball is weak-* compact.
f. $X = X^{**}$.

2. Give an example of a function of unbounded variation.

3. Let's say your linear vector space X comes equipped with an inner-product. Define a norm on X from this inner product and show it really is one.

4. Give examples of Cauchy sequences that do and do not converge.

5. Prove that the space of sequences with finite support (finite number of non-zero elements) is not complete. What is its completion?

6. Compute the Fourier series of the square wave x(t) that equals 1 for t in [-a, 0] and –1 for t in [0, a], and explain its significance.

7. Show that a closed convex set K in a normed linear space is the intersection of all the closed half spaces containing it.

8. Consider the space $L_2[-1,1]$. Orthogonalize the set {x^2, x^3, x^4}. Do not bother to normalize the vectors. That is, get a set of three orthogonal vectors {z1, z2, z3} which have the same span as the span of {x^2, x^3, x^4}.

9. Project the vector x = (1, 2, 3, 4) onto the vector y = (4, 3, 2,1), i.e., find the closest point to x in [y], the span of y. Now pick another vector z that is independent of y and find the closest point to x in the span [y, z]. How should you pick z to make this calculation easier and why?

10. Suppose x(0) =1. Given a continuous function b on [0, 1], find a control u that moves x so that x(1)=0 so as to minimize the integral of |u| from 0 to 1, where dx/dt = b(t) u(t).