Mathematics 116
Convexity and Optimization with Applications

Assignment VI — Due in class on Tuesday, March 22.

Reading — Study chapter 5 of Luenberger's OVSM. For more on Least Squares, glance at chapter 4 (working through it could make a good project).

Exercises — From OVSM §5.14: #4, #6, #10, #11, #15, #16 (Does this last problem remind you of anything important from multivariable calculus?).

Writing — In addition to handing these few paragraphs in with the other problems, you may also post answers to the discussion section of our website.

1. Consider the step function *x(t)* that equals 1 for $-\pi \le t < 0$ and equals 0 for $0 \le t \le \pi$. Show that its Fourier series is $\frac{1}{2} + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{((-1)^n - 1)}{n}\sin(nt)$. Discuss what this means. I.e., in what sense does this converge? In $L_2[-\pi,\pi]$? Pointwise? Uniformly? Why? What is the significance of the constant term? Why are the coefficients of the cosine terms all zero?

2. Define a new inner product on $L_2[0, 1]$ by setting

$$(x \mid y) = \int_0^1 e^t x(t) y(t) dt.$$

Using this "weighted" inner product to define a norm, reconsider Example 1 on p. 66 of OVSM and find the control voltage function *u* that solves the motor moving problem given there but minimizes this norm. Explain what effect this change has on the previous energy minimizing solution.

Discussion — Please come to sections prepared to discuss the following questions. If you want to post your answers on the web site, please do so by Sunday.

1. For a sequence $\{x_n\}$ in $L_2[-\pi,\pi]$, we have seen three different ways we could say it converges: pointwise, uniformly, and in the L_2 norm (sometimes called convergence in mean). Give arguments and examples that illustrate the relations between these notions. E.g., show that uniform convergence implies pointwise convergence, but give an example to show the converse is not true, then show that pointwise convergence does not imply L_2 convergence, etc.

2. Euler showed that 1 + 1/4 + 1/9 + 1/16 + …. converges to $\pi^2/6$. This is not what he did, but try to derive this by computing the Fourier series of x(t) = t in $L_2[-\pi,\pi]$, then use Parseval's Identity (Problem 16 of §4.13 as done in class). Can you show that $\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90}$? Others?