CHECKLIST 1. Week lecture 2/7/2005 — Math118, O.Knill

Checklist lecture of 7. January, 2005. The two main points are that you

- know how to **check stability of periodic points**. The absolute value of the derivative of $f^n(x)$ at the fixed point is relevant.
- **be familiar with two bifurcation scenarios**: the saddle-node (blue-sky) and the flip (pitch-fork) bifurcation. The later is important for the period doubling scenario in the logistic map. The first one occurs in the logistic map, when a period 3 orbit is created.

- **Logistic map:** A map of the form $T(x) = cx(1-x)$ is called a **logistic map**. It maps the interval $[0,1]$ onto itself if $0 \leq c \leq 4$.
- **Orbits:** We try to understand orbits of T, especially **Periodic orbits** which are also called **cycles**.
- **Fixed points**. Fixed points of T^n are periodic orbits of period n.
- **Cobweb construction**. A graphical way to visualize orbits. (demo).
- **Stability**. A fixed point x is stable if $|T'(x)| < 1$. For a stable orbit, a neighborhood of initial conditions converges to the fixed point. A periodic orbit of period n is stable, if it is a stable fixed point of T^n.
- **Bifurcation point**. A parameter point c at which the stability or the number of periodic points changes is called a **bifurcation point**. Necessarily, we have $|(T_c^n)'(x)| = 1$.
- **Period doubling bifurcation**. A stable periodic point becomes unstable and two periodic orbits of a period twice as large appear. At such a bifurcation point, we have $(T_c^n)'(x) = -1$. The period doubling bifurcation is a **flip bifurcation** for T_c^2.
- **Feigenbaum universality**. The successive period doubling bifurcation points c_k define a number $\lim_{k\to\infty} \frac{c_{k+1}-c_k}{c_{k+2}-c_{k+1}} = \delta = 4.69920166$ which is universal. The same constant appears for other maps also like $g_c(x) = c\sin(\pi x)$.
- **Saddle node bifurcation**. A saddle node bifurcation occurs, when the graph of f_c is tangent to the diagonal from one side. At one side of the bifurcation parameter, the graph has no intersection with the diagonal. At the other side of the bifurcation parameter, the graph has two intersections.

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1.1 We consider the interval map $f(x) = f_4(x) = 4x(1-x)$. For this particular value, the logistic map is also called the **Ulam map**. We have met it in the first lecture, when we saw that a computer does not "know" the distributative law.

a) Find all the fixed points of the map $T(x) = f(x)$.

b) Analyze the stability of these fixed points. For each point, just tell, whether it is stable or unstable.

c) Draw a graph of this map and start iterating the map using the cobweb construction with the initial value 0.3. Do at least 5 iterates.

1.2 Consider the map $Q_c(x) = x^2 + c$. It is called the **quadratic map**. Again, c is a constant parameter.

a) Verify that this map undergoes a **saddle node bifurcation** (which is also called **blue-sky bifurcation** because of obvious reasons, periodic appear or disappear out of the blue sky. It is also called **tangent bifurcation**). For which value of c does this happen?

b) Analyze the stability of the periodic orbits near the bifurcation value.

c) What happens with the orbits for parameter values c for which we have no fixed point?

1.3 a) Look at the fixed points of the map $Q_c(x) = x^2 + c$ for $c = -0.5$ and determine their stability.

b) Look at the fixed point of the map for $c = -1$ and determine its stability.

c) Verify that the map undergoes a **flip bifurcation** at the parameter $c = -3/4$.