CHECKLIST Lecture of 2/9/2005 — Math118, O.Knill

Checklist lecture of January 9th, 2005. (Some of the material might only be done on January 11'th depending on how much discussion we do on Wednesday). We introduce the Lyapunov exponent of an orbit as the exponential growth rate of $(f^n)'$.
We can compute the Lyapunov exponent in the case of periodic orbits and in the case of the tent map.
Then, we show that the tent map is conjugated to the Ulam map $f_4(x) = 4x(1-x)$.

- know the definition and the meaning of the Lyapunov exponent and how one computes the Lyapunov exponent.
- understand the relation between Lyapunov exponents and stability in the case of periodic orbits.
- see the conjugated maps have the same dynamics and the same Lyapunov exponent
- see the conjugation in the concrete situation of the Ulam and tent map.

Remark to a definition which we might not have much time to discuss this week:
The **dynamical zeta function** of f is defined as

$$\log(\zeta_f(z)) = \sum_{n=1}^{\infty} \frac{P_n(f)}{n} z^n ,$$

where P_n is the number of periodic points of f with period n. One can compute the dynamical zeta function of the Ulam map using the result shown in [1.5]. This leads to a closed formula.
(Note that $d/dz \log(\zeta_f(z))$ is $\sum_{n=1}^{\infty} \frac{P_n(f)}{z}^{n-1}$ which is $\sum_{n=1}^{\infty} \frac{P_n(f)}{z}^{n}$. Use the formula for the geometric sums $\sum_{n=1}^{\infty} a^n = 1/(1-a)$.)

It might be a bit strange why one should consider this function. It was introduced by Artin-Mazur in 1965 and Smale in 1967. For some dynamical systems like the tent map one can explicitely compute the function. There are many reasons, why one wants to study this object, algebraic reasons, there are connections with statistical mechanics and the **topological entropy** which is defined as

$$p(f) = \lim_{n\to\infty} \frac{1}{n} \log |P_n(f)| .$$

HOMEWORK 1. Week lecture 2/9/2005 — Math118, O.Knill

This is the second part of the homework. All the five problems [1.1] until [1.5] are due next Monday the 14th of february

1.4 We define the map $f(x) = 5x + \sin(\pi x) \bmod 1$ on the interval $[0,1]$.

Sideremark: Because after identifying 0 and 1, the interval closes to a circle, the map can be considered a smooth map on the circle. f is an example of a **circle map**.

a) What is the Lyapunov exponent of the orbit of the map f wich an initial condition $x_0 = 1/2$?

b) Verify that the Lyapunov exponent of every orbit of f is positive.

1.5 We have shown that the **Ulam map** $f_4(x) = 4x(1-x)$ is conjugated to the tent map $T(x) = 1 - 2|x - 1/2|$

a) Draw the graph of the the iterates $T^2(x), T^3(x)$ of the tent map. Use the fact that the tent map is piecewise linear.

b) Use the conjugation result to sketch the graphs of the second iterate $f_4^2(x)$ and third iterate $f_4^3(x)$ of the Ulam map.

c) Conclude that f_4^n has 2^n fixed points and therefore, that the Ulam map f has 2^n periodic points of period n.

d) What is the Lyapunov exponent of a periodic point of period n?