

## PROJECTS

Math118, O. Knill

ABSTRACT. We give you project guidelines and show a list of projects. The format is quite free. An important part of the project to choose a topic, to choose a format, to gauge whether it is reasonable or not. The project has to do with one of the topics we covered in class.

### 1 Project guidelines

PROJECT FLAVOURS:

- Read an article and summarize its content. Check first, whether the mathematics is manageable. We have articles of all difficulty levels.
- Explore some dynamical system with the computer. You probably should be familiar with the programming language, or computer algebra system, you use.
- Review a book on dynamical systems of your choice. You probably should have looked at the book already by now.
- Understand and reformulate a proof of a difficult theorem. Examples are given below.
- Work on an unsolved problem and solve it (-). More seriously: it is enough to write an exposition about the problem, or do some experiments which make it plausible to the reader that there is something interesting going on.

SCOPE. Plan 2 days of intense work on the project. Chose your topic so that you can finish it. Gauging whether the project is realistic is part of the task. So, choosing a computer project only makes sense, if you have experience with the software, you want to use. Choosing a book review only is realistic if you have read part of the book. If you choosing a paper to summarize, it is good if it connects to some of your interests or other course work. Beside the projects presented here, you are free to choose anything on your own, as long as it is tied to dynamical systems.

DEADLINE. The project is due on May 21 2005. It can be handed in earlier. It is advisable to show me the project before handing it in. Start to write early! Directly type in your notes like a diary and polish at the end. Time management can be one of the biggest challenges in any project.

FORM. I strongly recommend  $\LaTeX$ , especially for mathematical content. There is a template on the course web-site. The use of Latex might slow you down for a few hours at first, but you will make up any minute later on. All our notes for this course have been written in  $\LaTeX$ .

$\LaTeX$  also makes it easy to structure the paper. References and referals are taken care of automatically, the structure with title, abstract etc are all prewired.

GRADING. There will be 4 grades for each paper: originality, correctness, style and presentation. Unlike in SAT essays (NYT article of today), length is pretty irrelevant. The originality does not mean that the paper has to contain an original result. An original thought, an original experiment, an original question can give full score. The presentation and style components not only looks at the form of the paper, illustrations etc. but also how easy it is to read and how attractive the paper is overall.

LENGTH. Anything between 2-10 pages is ok. One strategy is to write a lot, then compress and trim things. Mark Twain once said:

"I didn't have time to write a short letter, so I wrote a long one instead."

### 2 Project suggestions: a bazaar

CONTINUED FRACTIONS. Continued fractions and the Gauss map. Give a proof that  $dx/(1+x)$  is the invariant measure of the Gauss map on the unit interval. Then prove Proposition 15.3.3 - 15.3.5 in the book of Katok-Hasselblatt.

QUATERNIONIC JULIA SETS. State definitions of the analogue of the Mandelbrot set or Julia set when iterating maps on the quaternions instead of the complex numbers. The quaternionic fractals are objects in 4D and usually presented as three dimensional slices. Explain the problem and make some pictures.

DIFFERENTIAL NEWTON METHOD. The differential Newton method. Read research paper of John Neuberger which had appeared in the Intelligencer. Unlike in the discrete case, the basins of attraction have smooth boundaries.

INTEGRABILITY IN DYNAMICAL SYSTEMS. Look at Audins article on integrability in dynamical systems.

IRRATIONAL GROWTH IN LIFE. Study irrational life. Read the article of William Geller and Michael Misiurewicz on irrational life.

THE FEIGENBAUM STORY. The Feigenbaum fixed point. What is renormalization. What does the theorem say? There is Mathematica code by Marek Rychlik.

DISCRETE APPROXIMATIONS OF CHAOTIC MAPS. Look at Lanfords article on orbit structure of discrete approximations to chaotic maps. This could lead to your own experiments with discretizations.

THE LATEST ABOUT THE LORNTZ ATTRACTOR. What is known about the Lorentz attractor. Read and understand the article of Viana.

VIRAL DYNAMICS. Read some papers on Viral infections or a chapter of Novaks book.

CELLULAR AUTOMATA WITH RULES DEPENDING ON MACROSCOPIC VARIABLES. Investigate experimentally a cellular automata, where the CA rule changes depending on global properties. Take for example life with three different rules and switch rule if the global density changes.

PERIODIC POINTS OF BILLIARDS. The problem of periodic points in billiards appears in different places. The case of triangular billiards where one does not know whether there are always periodic points or the case of smooth billiards where one does not know whether there can be sets of positive measure. The project could also include exterior billiards.

STABILITY OF EXTERIOR BILLIARD. Exterior billiards in semi circle. Make some experiments and form an opinion whether the table is stable.

INDECOMPOSABLE CONTINUA IN DYNAMICAL SYSTEMS. Read a paper Judy Kennedy, "How indecomposable continua arise in dynamical systems" and how it ties in with our course.

WOMEN IN DYNAMICAL SYSTEMS. Write an expository article about contributions of women in dynamical systems theory. In each example, the relevant mathematics should be described. Examples: Sonja Kovalevskaya, Mary Cartwright, Krystina Kuperberg, Emmy Noether, Lai-Sang Young, Bodil Branner, Erika Jen, Linda Keen, Jane Cronin Scanlon, Michele Audin.

WAVE DYNAMICS IN RELATIVISTIC SETUP. General relativity in 1+1 dimensions. How can wave fronts look like in an inhomogeneous medium, where the distance is given by a metric with signature  $+-$ .

THE CODING OF THE CAT MAP. Write a careful exposition about the coding in the cat map  $T(x, y) = (2x + y, x + y)$ . Can one do the encoding faithfully with three sets as we have seen in the homework?

CONTINUED FRACTIONS AND MUSIC. Continued fractions in music. The article had been distributed in class.

CHAOS IN THE SOLAR SYSTEM. Read and summarize the paper of Kirchgraber and Stoffer: Chaotic motion proof of comets. There are also some books about chaos in the solar systems which could be part of the project.

THE  $3n+1$  PROBLEM.  $3n+1$  problem. Exclude certain periodic patterns using Mathematica. Example:  $(3x + 1)/4 = x$  implies  $x = 1$ . The problem to relate cycle length with where the cycle is is related to continued fraction expansion of  $\log(3)/\log(2)$ .

A LATTICE POINT PROBLEM. Study the map  $T(x) = (3/2)x \bmod 1$ . This problem is a lattice point problem for a function on the real line. It is conjectured that the orbits of  $T$  are uniformly distributed modulo 1. Run some statistical experiments to check this and find out what is known about it.

THE KAM THEORY. KAM theorem. Track down what the KAM theorem is and where it is used. A possibility is to focus on the twist map theorem and see what it means for maps like the Standard map, billiards or exterior billiards or some elliptic periodic points.

WAVE FRONTS IN 3D. Visualize wave fronts in 3D emanating from a two dimensional surface. Find examples, where one knows the caustic as in 2D. To plot surfaces, start with a parametrized surface, draw the normals and plot all points in a distance  $d$  from the surface. This can be a computer graphics project. A more mathematical task would be to find the equation for the caustics and visualize the caustic.

POINCARÉ'S BLUNDER. Formulate the mathematics of the prize problem which King Oscar II of Sweden had posed. There is a Book of June Barrow-Gree, Poincaré and the three body problem, AMS 1996 you can borrow.

ARNOLD'S THEOREM IN INTEGRABLE SYSTEMS. Arnold's theorem on integrability. Write down a proof of Arnold's theorem telling that if a Hamiltonian system of  $n$  degrees of freedom has  $n$  independent involutive integrals, bounded solutions must lie on tori.

REAL QUADRATIC MAP SURVEY. Summarize Lubich's survey article on real quadratic maps.

DYNAMICAL SYSTEMS SURVEY. Summarize one of Lai-Sang Young's survey articles on dynamical systems.

SHAR KOVSKI THEOREM. What does it say? There is a nice proof in the book of Brin-Stuck.

HOMOCLINIC TANGLE IN THE STANDARD MAP. Visualize the homoclinic tangle in the Standard map. Adapt the code for the Henon map to the Standard map.

DOUBLE PENDULUM SYSTEM. Experiment with the double pendulum system. Mathematica code is available. Plot some Poincaré sections which are area preserving maps. Alternatively, one could look at nonlinearly coupled penduli.

MANDELBROT SET. HISTORY AND OTHER SETS. Write down the story of the discovery of the Mandelbrot set. Alternatively, look at other Mandelbrot sets like the map  $f_c(z) = z^2 \sin(z) + c$ .

BETA EXPANSION. Explore the invariant measures of the beta expansion. (Book of Dajani and Kraaikamp).

NEWTON PROBLEM WITH CONSTANT INTERACTION FORCE. The Newton problem in one dimension. Simulate 3 particles on the line with constant force interaction. This is a model for three galaxies.

IS  $\pi$  NORMAL. Find out what's about this recent indication that  $\pi$  is not normal. Alternatively, there is an article of Bailey, Borwein, Borwein and Plouffe to read. The paper should be available now Journal of Modern Physics C, vol. 16, no. 2.

FIXED POINTS IN DYNAMICAL SYSTEMS. Write an essay on "fixed points in dynamical systems". Particularly interesting is Poincaré's last theorem which is mentioned in the book as well as Brouwer's fixed point theorem or the Newton method which is used to prove the existence of the Feigenbaum fixed point.

COMPLEX HENON MAPS. Read an article on Complex Henon maps and state some questions one can ask when iterating maps in the two dimensional complex space  $C^2$ .

LYAPUNOV EXPONENTS OF HENON MAP. Lyapunov exponents of Henon map <http://alamos.math.arizona.edu/rychlik/notebooks.html>

LOZI MAPS. Lozi maps are Henon type maps, which are piecewise linear. They are easier accessible.