4/18 QUIZ NR 10	**Math118, O.Knill**

Name:

1) What is special about the **golden ratio** θ? Check everything which applies:

a) It has the smallest possible continued fraction expansion $\theta = [a_0; a_1, a_2, a_3, ...]$.
b) The partial fractions p_n/q_n have the property that q_n and p_n grow like Fibonnacci numbers.
c) It satisfies $x = 2/(2 + x)$.

2) Which of the the following numbers has the continued fraction expansion

$$x = [0; 2, 1, 2, 1, 2, 1, ...] = \frac{1}{2 + \frac{1}{1+\frac{1}{2++....}}}$$

a) x satisfies $x = 1/(2 + 1/(1 + x))$.
b) x satisfies $x = 1/(2 + 1/x))$.
c) x satisfies $x = 1/(1 + 2/x))$.
d) x satisfies $x = 1/(1 + 1/2 + x))$.

3) Which of the following statements is called Chebychevs theorem for an irrational number α.

a) For every n, there exists $q \leq n$ such that

$$|\alpha - p/q| \leq 1/q^2 .$$

b) For every n, there exists $q \leq n$ such that

$$|\alpha - p/q| \leq 1/q .$$

c) For every q, there exists p such that

$$|\alpha - p/q| \leq 1/q^2 .$$

d) For every q, there exists p such that

$$|\alpha - p/q| \leq 1/q .$$

4) Which of the following three formulas give the correct recursion for the partial fractions p_n/q_n:

a) $p_{n+1} = a_n p_n + p_{n-1}$, $q_{n+1} = a_n q_n + q_{n-1}$.
b) $p_{n+1} = p_n + a_n p_{n-1}$, $q_{n+1} = q_n + a_n q_{n-1}$.
c) $p_{n+1} = a_n p_n + a_{n-1} q_{n-1}$, $q_{n+1} = a_n p_n + a_{n-1} q_{n-1}$

5) The dynamical logarithm problem is the problem

a) to find the time to reach from a point x to a neighborhood of a point y.
b) to find the point y which is reached after time t when starting from x.
c) to find the initial point x, when reaching the point y after time t.

6) Which dynamical system is involved when making a decimal expansion of a real number.

a) $T(x) = 10x$.
b) $T(x) = 10x \bmod 1$.
c) $T(x) = 10/x \bmod 1$.

7) The quadratic map $T(x) = x^2 + c$ is also useful in number theory. Where?

a) to compute the eclipse times in calendars.
b) to factor large integers.
c) to understand why our tonal system has 12 scales between an octave and 19 scales for a perfect fifth .

8) Who came up with the idea to factor integers n by finding two numbers x and y satisfying $x^2 = y^2 \bmod n$ and then having a common nontrivial factor of $x - y$ with n?

a) Fermat.
b) Tchebychev.
c) Minkovsky.

9) When finding lattice points close to graphs of quadratic polynomials, we were led to a dynamical system on the two dimensional torus. This system is

a) $(x, y) \to (x + 2a, x + y) \bmod 1$.
b) $(x, y) \to (2x + y, x + y) \bmod 1$.
c) $(x, y) \to (x + y, x - y) \bmod 1$.
d) $(x, y) \to (ax^2 + bx + c, x) \bmod 1$.

10) Check for whatever continued fractions are useful:

a) To compute eclipse cycles.
b) To justify why we use a 12 scale system in music.
c) To find lattice points close to lines in the plane.
d) To factor integers.