3/14/15926... QUIZ NR 6 **Math118, O.Knill**

Name:

1) The billiard in an ellipse is known to be integrable. What is the integral F?
a) The sum of the shortest distances of the trajectory line to the focal points.
b) The product of the shortest distances of the trajectory line to the focal points.
c) The impact angle θ.
d) The distance of the focal points.

2) A billiard table γ is obtained by doing the string construction to a convex set K. (For example, in the homework for today, K was a triangle or square):
Check all which applies:

a) The billiard has a caustic.

b) The billiard map can not have glancing trajectories: trajectories for which the angle θ can become arbitrarily close to 0 and arbitrarily close to π. .

c) The billiard map has periodic orbits of period 17.

d) The billiard map has an invariant curve in the annulus $R/Z \times [-1, 1]$.

3) For which coordinates is the billiard map area-preserving?

a) The (s, θ) coordinates, where s is the arc length normalized that the table has length 1 and where θ is the impact angle.

b) The (x, y) coordinates, where x is the arc length normalized so that the table has length 1 and where $y = \cos(\theta)$.

c) The (s, s') coordinates, where (s, s') are successive impact points of the trajectory and where s is the arc length parameter.

4) Every strictly convex smooth Birkhoff billiard has periodic orbits, because

a) We can maximize the length functional of the polygon.

b) We can minimize the length functional of the polgyon.

c) We can maximize the area functional inside a polygon.

d) We can minimize the area functional inside a polygon.

5) Which of the following are open mathematical problems?

a) Every billiard in a triangle has a periodic orbit.

b) Every exterior billiard has the property that for (x, y) outside the table, $T^n(x, y) \to \infty$.

c) The solar system is stable in the sense that all planets remain in a bounded region near the sun for all times.

d) There exists a convex billiard for which the Lyapunov exponent is positive on a set of positive area.

e) There exists a smooth convex billiard for which there are no glancing orbits.

6) Which of the following equations is called the **Euler equation**? We use the notation $h_1(x, y) = \frac{\partial}{\partial x} h(x, y)$ and $h_2(x, y) = \frac{\partial}{\partial y} h(x, y)$. Just one answer is correct.

a) $h_1(x_{i-1}, x_i) + h_2(x_i, x_{i+1}) = 0$.

b) $h_2(x_{i-1}, x_i) + h_1(x_i, x_{i+1}) = 0$.

c) $h_1(x_{i-1}, x_i) + h_1(x_i, x_{i+1}) = 0$.

d) $h_2(x_{i-1}, x_i) + h_2(x_i, x_{i+1}) = 0$.

e) $h_1(x_{i-1}, x_i) - h_2(x_i, x_{i+1}) = 0$.

f) $h_2(x_{i-1}, x_i) - h_1(x_i, x_{i+1}) = 0$.

g) $h_1(x_{i-1}, x_i) - h_1(x_i, x_{i+1}) = 0$.

h) $h_2(x_{i-1}, x_i) - h_2(x_i, x_{i+1}) = 0$.

7) Which of the following matrices is conjugated to the Jacobean $DT(x_i, y_i)$ of the billiard map. l_i is the length of the trajectory before the impact with the boundary where the impact angle is θ_i and the curvature is κ_i.

a) $B_i = \begin{bmatrix} 1 & 0 \\ -\frac{2\kappa_i}{\sin(\theta_i)} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & l_i \\ 0 & 1 \end{bmatrix}$

b) $B_i = \begin{bmatrix} 1 & l_i \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -\frac{2\kappa_i}{\sin(\theta_i)} & 1 \end{bmatrix}$

c) $B_i = \begin{bmatrix} 1 & 0 \\ \frac{2\kappa_i}{\sin(\theta_i)} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -l_i \\ 0 & 1 \end{bmatrix}$.

8) Today is Pi-day. Somebody cuts a piece from a circular apple pie. We use the remaining part as a billiard table. Which of the following is true:

a) The table is convex.

b) The table is not convex.