§2.6 #4 (continued)

$$a - 2b = 1 \quad \cdots (5)$$
$$a + b + c = 0 \quad \cdots (6)$$
$$b - 3c = 0 \quad \cdots (7)$$

Substituting (7) into (5) and (6) we find that $a - 6c = 1$ and $a + 4c = 0$

and therefore that $c = -1/10$ and $a = 2/5$. From (7), we find that $b = -3/10$, and so $x_1 = (2/5, -3/10, -1/10)$.

Similarly, solving $f_1(x_2) = 0 \quad f_2(x_2) = 1 \quad f_3(x_2) = 0$

we find that $x_2 = (3/5, 3/10, 1/10)$

and solving $f_1(x_3) = 0 \quad f_2(x_3) = 0 \quad f_3(x_3) = 1$

we find that $x_3 = (1/5, 1/10, -3/10)$.

The basis dual to $\{f_1, f_2, f_3\}$ is therefore

$$\{(2/5, -3/10, 1/10), (3/5, 3/10, 1/10), (1/5, 1/10, -3/10)\}$$

§2.6 #6

(a) $$(T^t f)(x,y) = f(T(x,y)) = f(3x+2y, x)$$
$$= 2.(3x+2y) + x$$
$$= 7x + 4y$$

(b) Since $\beta^* = \{f_1, f_2\}$ is the dual basis to $\beta = \{(1,0), (0,1)\}$

we have $f_1(x,y) = x$
$f_2(x,y) = y$

Now $$(T^t f_1)(x,y) = f_1(T(x,y))$$
$$= f_1(3x+2y, x) = 3x + 2y$$