The matrix $[T_w]_\beta^\beta$ is

$$[T_w]_\beta^\beta = \begin{array}{cc} & \overbrace{\hphantom{0 \quad 0 \quad \cdots \quad 0}}^{k-1} \\ k \left\{ \vphantom{\begin{matrix} 0\\1\\0\\ \vdots \\ 0 \end{matrix}} \right. & \begin{pmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & & 0 & -a_1 \\ 0 & 1 & & 0 & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -a_{k-1} \end{pmatrix} \end{array}$$

because e.g. for the first column:

$$T(v) = 0.v + 1.T(v) + 0.T^2(v) + \cdots$$

and for the last column

$$T(T^{n-1}(v)) = -a_0 v - a_1 T(v) - \cdots - a_{k-1} T^{k-1}(v)$$

This lets us compute the characteristic polynomial $f(t)$ of T_w:

<u>Claim</u> (cf Th$^{\underline{m}}$ 5.22 (b))

$$f(t) = (-)^k (a_0 + a_1 t + \cdots + a_{k-1} t^{k-1} + t^k)$$