

example

Suppose that $\beta = \{v_1, \dots, v_a, w_1, \dots, w_b, \dots, z_1, \dots, z_c\}$ is ~~the~~^a basis with respect to which $[T]_{\beta}^{\beta}$ is in Jordan canonical form:

$$[T]_{\beta}^{\beta} = \begin{matrix} & \begin{matrix} \xrightarrow{a} & \xrightarrow{b} & \dots & \xrightarrow{c} \end{matrix} \\ \begin{matrix} \downarrow a \\ \downarrow b \\ \dots \\ \downarrow c \end{matrix} & \begin{pmatrix} A_1 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & A_k \end{pmatrix} \end{matrix}$$

Then the "first block" $\text{span}\{v_1, \dots, v_a\}$ is

T -invariant: if $v = \alpha_1 v_1 + \dots + \alpha_a v_a$ then

$$[T(v)]_{\beta} = [T]_{\beta}^{\beta} [v]_{\beta}$$

$$= \begin{matrix} \begin{matrix} \downarrow a \\ \downarrow b \\ \dots \\ \downarrow c \end{matrix} & \begin{pmatrix} A_1 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & A_k \end{pmatrix} & \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_a \\ 0 \\ \vdots \\ 0 \end{pmatrix} \end{matrix}$$

$$= \begin{pmatrix} A_1 v \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{matrix} \leftarrow \text{vector of size } a \\ \leftarrow \text{rest are zeroes} \end{matrix}$$