#7 (a) If T is nilpotent, say $T^k = 0$, and $Tx = \lambda x$ then
$$\begin{aligned} 0 &= T^k(x) \\ &= \lambda^k \{x\} \end{aligned}$$

$\Rightarrow \lambda = 0$. So every eigenvalue of a nilpotent linear operator is zero.

Conversely, if every eigenvalue of T is zero then for a Jordan canonical basis β $[T]_\beta^\beta$ is upper-triangular with zeroes on the diagonal. Thus $([T]_\beta^\beta)^n = 0$, where $n = \dim V$, so $T^n = 0$.

(b) If $A = [T]_\beta^\beta$ and $B = [T]_\gamma^\gamma$ where β and γ are different ~~matrices~~ bases for V then $B = Q^{-1}AQ$ where $Q = [I_d]_\gamma^\beta$

Thus, by Q1(a), $tr(B) = tr(A)$.

(c) Choose a basis β for V such that

$[T]_\beta^\beta$ is upper-triangular

(for example, β could be a Jordan