Then $V \oplus W = \mathbb{R}^3$

and $V \oplus Z = \mathbb{R}^3$ but $W \neq Z$

(d) This is true. To show linearity, I need to show that:

$(\star)$ $T(p+q) = T(p) + T(q)$ for all $p, q \in P(\mathbb{R})$

$(\star)$ $T(cp) = cT(p)$ for all $p \in P(\mathbb{R})$ and all $c \in \mathbb{R}$

$$T(p+q) = (p+q)(x^2)$$
$$= p(x^2) + q(x^2)$$ definition of addition of functions
$$= T(p) + T(q)$$

and
$$T(cp) = (cp)(x^2)$$
$$= c.\,p(x^2)$$ definition of scalar mult. of functions
$$= c.\,T(p)$$

So T is linear.

Q2 (a)

$\beta = \{1, x, x^2\}$

$\gamma = \{1, x, x^2, x^3\}$

$T(1) = x = 0.1 + 1.x + 0.x^2 + 0.x^3$

$T(x) = \frac{1}{2}x^2 = 0.1 + 0.x + \frac{1}{2}x^2 + 0.x^3$

$T(x^2) = \frac{1}{3}x^3 = 0.1 + 0.x + 0.x^2 + \frac{1}{3}.x^3$