<u>Claim</u> : $N(S) = W$

<u>Proof</u> : $v \in N(S) \iff v + W = 0_{V/W}$

$\iff v + W = 0_V + W$

$\iff v - 0_V \in W$

$\iff v \in W$. $\square$

<u>Claim</u> : $\dim (V/W) = \dim V - \dim W$

<u>Proof</u> : The dimension theorem gives

$$\dim V = \dim R(S) + \dim N(S)$$
$$= \dim V/W + \dim W. \quad \square$$

(b) <u>Claim</u> : $\overline{T} : V/_{N(T)} \longrightarrow R(T)$

is (i) ~~well-defined~~ onto
(ii) an isomorphism

<u>Proof</u> : (i) Given $w \in R(T)$, we can find $v \in V$ with $T(v) = w$.
But then $\overline{T}(v + N(T)) = T(v) = w$

so $\overline{T}$ is onto.