(ii) $\dim V/N(T) = \dim V - \dim N(T)$

and $\dim R(T) = \dim V - \dim N(T)$

(Dimension Theorem!)

so $\overline{T}$ is an isomorphism $\iff$ $\overline{T}$ is onto.
But we know $\overline{T}$ is onto from (i). $\square$

4. <u>Claim</u>: $R(T)^\circ = N(T^t)$

<u>Proof</u>:

$$\begin{aligned} f \in R(T)^\circ &\iff f(y) = 0 \quad \text{for all } y \in R(T) \\ &\iff f(T(x)) = 0 \quad \text{for all } x \in V \\ &\iff T^t(f)(x) = 0 \quad \text{for all } x \in V \\ &\iff T^t(f) = 0_{V^*} \\ &\iff f \in N(T^t). \end{aligned}$$ $\square$

T is diagonalizable if and only if both the following conditions hold:

(a) the characteristic polynomial of T splits

(b) for each eigenvalue $\lambda$ of T, $\dim E_\lambda = \text{mult}(\lambda)$

eigenspace for $\lambda$

algebraic multiplicity of $\lambda$

<u>Claim</u>: The characteristic polynomial of T is equal to the characteristic polynomial of $T^t$.

<u>Proof</u>: char. poly of $T = \det([T]_\beta^\beta - tI)$

where $\beta$ is some basis for V