$\boxed{7.2.10}$

(a) $\dim(K_\lambda) = \dim(N(T-\lambda I)^k) = n_1(\lambda) + 2n_2(\lambda) + \dots + k n_k(\lambda) = \sum_{i=1}^{n} i n_i(\lambda)$

where n_i repres the # of $i \times i$ blocks of eigenvalue λ, and i is the length of each block.

(b) i.e. $n_i(\lambda) = 0$ for $i > 1$.

$\Leftrightarrow \dim(\cancel{E_\lambda}) = \dim N(T-\lambda I) = n_1(\lambda)$
E_λ.

but $\dim(K_\lambda) = n_1(\lambda) = \dim(N(T-\lambda I)^k)$

$\Big\} \Leftrightarrow \dim(E_\lambda) = \dim(K_\lambda)$
Know E_λ subspace of K_λ $\Big| \Rightarrow$

$\Leftrightarrow \underline{E_\lambda = K_\lambda}$

$\boxed{7.2.12}$

Proceed by induction on n:

Base case $n=1$ is trivial.

Induction step $P(n) \Rightarrow P(n+1)$

Suppose hypothesis holds for any upper-Δ $n \times n$ matrix w/ zeroes on diagonal.

~~That~~ Let

$$A_{n+1} = \begin{bmatrix} 0 & a_{12} & & & \\ & 0 & a_{23} & & \\ & & \ddots & & \vdots \\ & 0 & & 0 & a_{n,n+1} \\ & & & & 0 \end{bmatrix} = \left[\begin{array}{c|c} A_n & \vec{a} \\ \hline 0 & 0 \end{array}\right]$$

for A_n an $n \times n$ matrix satisfying the above. $\vec{a}$ an $n \times 1$ vector

Then $\exists\, p \in \mathbb{N}$ such that $(A_n)^p = (0)$ by induction hypothesis.

But $A^{p+1} = \left[\begin{array}{c|c} (A_n)^{p+1} & \vec{a}(A_n)^p \\ \hline 0 & 0 \end{array}\right] \Rightarrow A^{p+1} = (0)$ and we're done!

$\boxed{7.2.17}$

(a) $[S]_\beta^\beta = \begin{bmatrix} \lambda_1 & & & & & \\ & \ddots \lambda_1 & & & & \\ & & \lambda_2 & & & \\ & & & \ddots \lambda_2 & & \\ & & & & \ddots & \\ & & & & & \lambda_k \end{bmatrix}$

where β = Jordan canonical basis for T.

So S is diagonalizable. ✓

(b) Nilpotence follows directly from #12 above; To show that it commutes simply perform matrix multiplication and result will follow.