FRONT & BACK.

$\lambda_1 = 3 \longleftrightarrow$ eigenvectors $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} t$, $t \in \mathbb{R}$.

$\lambda_2 = \lambda_3 = \lambda_4 = 1 \longleftrightarrow$ eigenvectors $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} t$, $\begin{bmatrix} -4 \\ 0 \\ 1 \\ 0 \end{bmatrix} u$, $\begin{bmatrix} -8 \\ 0 \\ 0 \\ 1 \end{bmatrix} v$, $t, u, v \in \mathbb{R}$.

$\beta = \{ x, -2 + x, -4 + x^2, -8 + x^3 \}$ - ordered basis

so: $[T]_\beta = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is a diagonal matrix.

(g) $A = [T]_\gamma = \begin{bmatrix} -1 & -2 & -2 & -8 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ where $\gamma = \{1, x, x^2, x^3\}$.

Eigenvalues & eigenvectors:

$\lambda_1 = -1 \longleftrightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} t$ $\quad\lambda_3 = 2 \longleftrightarrow \begin{bmatrix} 2 \\ 0 \\ -3 \\ 0 \end{bmatrix} t$

$\lambda_2 = 1 \longleftrightarrow \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} t$ $\quad\lambda_4 = 3 \longleftrightarrow \begin{bmatrix} -7 \\ 6 \\ 0 \\ 2 \end{bmatrix} t$, $t \in \mathbb{R}$.

So $\beta = \{1, 1-x, 2-3x^2, -7+6x-2x^3\}$ & $[T]_\beta = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

(h) $[T]_\gamma = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$, $\gamma = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

$\lambda_1 = \lambda_2 = \lambda_3 = 1 \Longleftrightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} t$, $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u$, $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} v$, $t, u, v \in \mathbb{R}$.

$\lambda_4 = -1 \longleftrightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} t$, $t \in \mathbb{R}$.