Math 122 / Problem Set 7

Written problems due Monday, November 7

Monday, October 31

1. Let G be a p-group, and let S be a finite set on which G operates. Assume that the order of S is not divisible by p. Show that there is a fixed point for the action of G on S (that is, an element $s \in S$ whose stabilizer is the whole group).

2. Determine the Class Equation of the dihedral group D_n of order $2n$.

3. Let Z be the center of a group G. Prove that if G/Z is a cyclic group then G is abelian and hence $G = Z$.

4. Let H be a subgroup of a group G. Prove or disprove: The normalizer $N(H)$ is a normal subgroup of the group G.

5. Let $H \subset K \subset G$ be groups. Prove that H is normal in K if and only if $K \subset N(H)$.

Reading: Artin §§6.4

Wednesday, November 3

6. How many elements of order 5 are contained in a group of order 20.

7. Let G be a group of order $p^e m$. Prove that G contains a subgroup of order p^r for every integer $r \leq e$.

8. Prove that if G has order $n = p^e a$ where $1 \leq a < p$, $e \geq 1$, and $(a, e) \neq (1, 1)$, then G has a proper nontrivial normal subgroup.

Reading: Artin §§6.5

Friday, November 5

9. Determine the the Class Equations of the groups of order 12.

10. Prove that a group of order $n = 2p$, where p is prime, is either cyclic or dihedral.

Reading: Artin §§6.6