

Homework 7

Math 124, Fall 2005

Due Wednesday, November 9nd

No late assignments will be accepted.

1. Show that it is impossible to draw an equilateral triangle with each of its vertices at lattice points. (Integer coefficients.) [Hint : We can put one of the vertices at the origin. We can assume that one of the coordinates is odd since otherwise we could take half this triangle. Now take congruences mod 4.]
2. Find all solutions to $x^2 + y^2 = 2z^2$. [Hint: Show that for a primitive solution both x and y are odd. Write $x + y = 2p$ and $x - y = 2q$.]
3. Find all solutions to $x^2 + 2y = 3z^2$. [Hint: Use the hint of the previous problem modulo 3.]
4. Show that $2x^2 + 5y^2 = z^2$ and $3x^2 + 5y^2 = z^2$ have no solutions in integers other than $(0, 0, 0)$.