MATH 126 PROBLEM SET 1: GROUPS

This problem set is due Wednesday September 27.

1) Show that $\operatorname{sgn}(12) = -1$ and that $\operatorname{sgn}(a_1a_2) = -1$ for all a_1, a_2. Also show that $(a_1a_r)(a_1a_2...a_{r-1}) = (a_1...a_r)$ and that $\operatorname{sgn}(a_1a_2...a_r) = (-1)^{r-1}$.

2) Show that $Z(G)$ is a normal subgroup of G.

3) Show that $D_{2n} \cong \langle a, b : \ a^n = b^2 = abab = 1\rangle$. (Pay attention to the details.)

4) Show that D_8 is nilpotent.

5) What are the orders of the various conjugacy classes in S_4? List all normal subgroups of S_4. Is S_4 solvable? Supersolvable?

6) Suppose $\sigma \in A_n$. If $C_{S_n}(\sigma)$ is not contained in A_n, show that $[\sigma]_{S_n} = [\sigma]_{A_n}$. If on the other hand $C_{S_n}(\sigma) \subset A_n$ show that $[\sigma]_{S_n}$ is the disjoint union of two A_n conjugacy classes of equal orders.

7) List all conjugacy classes in A_5 together with their orders. Use your list to find all normal subgroups of A_5.

8) Show that any abelian group is nilpotent.

9) Suppose that p is a prime number and that G is a finite group of p-power order which acts on a finite set Ω. Let $\Sigma \subset \Omega$ denote the subset of points fixed by G. Show that

$$\#\Sigma \equiv \#\Omega \bmod p.$$

10) If G is a finite group of prime power order show that $Z(G)$ is non-trivial. [Hint: Apply 9) to the action of G on itself by conjugation.] Deduce that G is nilpotent.