Math 126

First problem set

Due Sept. 23, 2003

1. Write the permutations

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 1 & 4 & 8 & 5 & 7 & 2 & 3 \end{pmatrix}$$

and

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 5 & 4 & 1 & 8 & 9 & 6 & 7 & 2 \end{pmatrix}$$

in cycle notation.

2. What is the order of each of the permutations in problem 1?

3. For any group G, the **conjugacy classes** are the orbits of G acting on itself via conjugation. Let D_4 be the group of Euclidean symmetries of the square. In other words, D_4 is the isotropy group $E(2)_\square$ where $\square$ is a square, say with center at the origin. So $\#D_4 = 8$: there are the rotations through $0^o, 90^o, 180^o$ and 270^o and four reflections: through the diagonals and and through the side bisectors. What are the conjugacy classes of D_4?

3. Let H be subgroup of a group G such that $\#H = \frac{1}{2}\#G$. Show that H is a normal subgroup of G.

4. Show that all groups G with $\#G = 15$ are isomorphic, and in fact isomorphic to the cyclic group $\mathbb{Z}/15\mathbb{Z}$. [Hint: Use the Sylow theorems to compute how many 3-Sylow subgroups and 5-Sylow subgroups there are. Use this information together with Lagrange's theorem to prove the existence of at least one element of order 15.]