Math 126, Midterm

Handed out 3 P.M., Friday, November 13, 1998.
Due at 2 P.M., Monday, November 16, 1998.

You may not work with others. You may use class notes and the course text, but no other resource. You may not consult the literature. You must sign and turn in this exam along with your clearly written and organized work. Write your name on every page you turn in. Good luck!

I certify that the work I am turning in for this midterm exam is entirely my own, that no one assisted me, and that I have abided by the rules above.

Signature:

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1. (50 pts.) Let $G = SL_2(F_3)$, the group of two by two matrices with entries in the finite field with three elements and determinant 1. Let $\bar{G} = G/\{\pm 1\}$. Let $\pi\colon G \to \bar{G}$ be the canonical projection.

1. Show that the order of G (resp. $\bar{G}$) is 24 (resp. 12).
2. Let $\bar{H} = \{\bar{g} \in \bar{G} \mid \bar{g}^2 = 1\}$. Show that $\bar{H}$ is normal in $\bar{G}$.
3. Let $I(G)$ and $I(\bar{G})$ denote the set of isomorphism classes of irreducible representations of G and $\bar{G}$ respectively. Show that $I(\bar{G})$ consists of four elements, namely, three one dimensional representations and one three dimensional representation.
4. Let $I^+(G)$ consist of those representations in $I(G)$ on which -1 acts as 1, and let $I^-(G)$ consist of those representations in $I(G)$ on which -1 acts as -1. Show that there is a natural bijection $I^+(G) \cong I(\bar{G})$.
5. Show that $\displaystyle\sum_{\rho \in I^-(G)} (\dim\, \rho)^2 = 12.$
6. Let $H = \pi^{-1}(\bar{H})$. Show that H has a unique irreducible representation μ on which -1 acts as -1.
7. Let $\rho \in I^-(G)$. Show that the restriction of ρ to H is isomorphic to a direct sum of two copies of μ. Conclude that $I^-(G)$ consists of three two dimensional representations.

2. (50 pts.) Do one of the following two problems:

1. Compute the character table for the symmetric group on 6 letters.
2. Compute the character table for the biicosahedral group.

You must show your work to get credit.