Math 126 Problem Set 1

This problem set is due October 2, 1998.

1. (25 pts.) Let M be an n by n matrix with complex entries and assume that there exists a positive integer N such that $M^N = 1$. Prove that M is diagonalizable. Find a counter-example if we drop the finite order assumption.

2. (25 pts.) Let R be a ring. Let E be a semisimple module over R. Prove that every submodule and every factor module of E is also semisimple.

3. (50 pts.) Let D_n denote the dihedral group of order $2n$. This is the group generated by two elements g and r which satisfy the relations: $g^n = 1$, $r^2 = 1$, and $rgr = g^{-1}$. Classify the simple representations of D_n. (Note that we've done the cases $n = 1$, 2, and 3 in class.)