MATH 152, FALL 2003
METHODS OF DISCRETE MATHEMATICS
Homework Assignment # 5
Due: October 21, 2003

Reading

- Read Biggs, Chapter 23, especially 23.1–23.4.
- Read the handout on Affine Geometry, up through the proof that addition is well defined.

Required Problems

1. Consider the quotient ring $R = \mathbb{Z}_3[x]/\langle q(x)\rangle$ where $q(x) = x^3 + x + 1$. How many elements are in R? Compute the products $[2x+1][x^2]$ and $[x+2][x^2+2x+2]$ in R.
2. Exhibit an isomorphism between the non-zero elements of $\mathbb{F}_8$ (that is, the multiplicative group of the field) and the elements of $\mathbb{Z}_7$ (considered as an additive group).
3. Use Euclid's gcd algorithm to find the greatest common divisor of $x^3 + 4x^2 + 2x + 2$ and $2x^3 + 4x^2 + 3$ as polynomials in $\mathbb{Z}_5[x]$. (The gcd is a quadratic polynomial.)
4. The polynomial $q = x^2 + 3x + 3$ is irreducible over $\mathbb{Z}_5$ and so can be used to construct the field $\mathbb{F}_{25}$. Use Euclid's algorithm to find the inverse of p $= [x + 2]$ in $\mathbb{F}_{25}$ by finding polynomials m and n such that $mp + nq = 1$.
5. Consider the quotient ring $R = \mathbb{Z}_5[x]/\langle q(x)\rangle$ where $q(x) = x^2 + 2$. How many elements are in R? Find the multiplicative inverse for the element $[ax + b]$, where $a \neq 0$. Find the orders of the elements $[x]$ and $[x + 1]$.
6. Biggs, problem 3 on page 317.
7. In the large affine faculty senate two "triangles" are Danny-Gavin-Viola (ABC) and Helen-Sally-Xenia ($A'B'C'$). Show that these six instructors satisfy the conditions and the conclusion of the Desargues axiom.

8. In the medium affine faculty senate, choose the library committee, with Jane as the additive identity. Add Greg and Mike, first using Kate as the auxiliary point, then using Irma. Draw a single diagram representing the addition with both auxiliary instructors, attaching a committee name to each line and an instructor name to each point. Identify the two "triangles" that are related by Desargues' theorem in the proof that the answer is independent of the choice of instructor.

Exploratory Problems

9. Consider the ring $R = F[x]$, where F is a field. Suppose $a(x), b(x) \in R$ are non-zero. Show that $deg(a(x)b(x)) = deg(a(x))deg(b(x))$. Construct a counterexample to show that this result is not always true in the ring $\mathbb{Z}_6[x]$.
10. Biggs, problem 5 on page 322.
11. (This is pasted in by hand because of the complicated diagram; so it will not appear in the Web version.)
12. (This is a second problem on the next page.)