MATHEMATICS 152, FALL 2003
METHODS OF DISCRETE MATHEMATICS
Outline #7a (Finite Affine Geometry – coordinates)

References

- Faculty Senate Affine Geometry, Theorem 21.
- Data files for the small, medium, and large affine senates.

This is an exercise for the entire class. To emphasize the point that the results are valid for any choice of origin and axes, several arbitrary choices have not been specified in advance.

1. Identify the blackboard with $\mathbb{R}^2$. The blackboard is to be regarded as an affine plane, not a Euclidean plane, so pay no attention to angles, and do not compare the lengths of segments on lines that are not parallel.

   (a) Choose an origin O.

   (b) Choose a line through O as the x-axis, and specify the multiplicative identity (1,0) on this line.

   (c) Choose another line through O as the y-axis, and specify the multiplicative identity (0,1) on this line. To avoid being influenced by concepts that are Euclidean rather than just affine, it it a good idea draw the y-axis so that it is not perpendicular to the x-axis and to choose the multiplicative identity for y at a different distance from O than the multiplicative identity for x.

   (d) Locate the point whose coordinates are (1,1) and the line that passes through (0,0) and (1,1).

   (e) Locate the point (2,2) and show how to use parallel lines to locate the points (2,0) and (0,2).

   (f) Choose an arbitrary point and show how to use parallel lines to associate coordinates (x, y) with it.

   (g) Consider the line that passes through (1,0) and (0,2). Show how to determine all the points on this line, both as a point plus an arbitrary multiple of a vector and as a linear equation satisfied by the coordinates of any point (x, y) on the line.

2. Associating a pair of coordinates from $\mathbb{Z}_3$ with every instructor in the small affine senate.

   (a) Choose any instructor to be the origin O.

   (b) Choose any committee through O as the x-axis, and specify the multiplicative identity ([1],[0]) on this committee.

   (c) Choose any other committee through O as the y-axis, and specify the multiplicative identity ([0],[1]) on this committee.

   (d) Identify the instructor whose coordinates are ([1],[1]) and the committee that passes through ([0],[0]) and ([1],[1]).

   (e) Identify the instructor ([2],[2]) and show how to use parallel committees to locate the instructors ([2],[0]) and ([0],[2]).

   (f) Choose an instructor whose coordinates are not yet determined, and show how to use parallel committees to associate coordinates (x, y) with this instructor.

   (g) Draw a diagram showing the nine instructors arranged in a square pattern with the x-axis horizontal and the y-axis vertical, showing the coordinates for each instructor.

   (h) Consider the committee that includes ([1],[0]) and ([0],[2]). Show how to determine all the instructors on this committee, both as a point plus an arbitrary multiple of a vector and as a linear equation satisfied by the coordinates of any point (x, y) on the line.

   (i) Determine which committee satisfies the equation $x + [2]y = [2]$.

3. Associating a pair of coordinates from $\mathbb{F}_4$ with every instructor in the medium affine senate. To avoid reusing the symbols x and y, call the elements of this field [0], [1], $[u]$, and $[u+1]$.

   (a) Choose any instructor to be the origin O.

   (b) Choose any line through O as the x-axis, and specify the multiplicative identity ([1],[0]) on this line.

   (c) Choose any other line through O as the y-axis, and specify the multiplicative identity ([0],[1]) on this line.

   (d) Identify the instructor whose coordinates are ([1],[1]) and the committee that passes through ([0],[0]) and ([1],[1]).

   (e) Choose the instructor $([u],[u])$ (this really does involve an arbitrary choice) and show how to use parallel committees to locate the points $([u],[0])$ and $([0],[u])$.

   (f) Choose an instructor whose coordinates are not yet determined, and show how to use parallel committees to associate coordinates (x, y) with this instructor.

   (g) Draw a diagram showing the sixteen instructors arranged in a square pattern with the x-axis horizontal and the y-axis vertical, showing the coordinates for each instructor.

   (h) Consider the committee that includes ([1],[0]) and $([0],[u])$. Show how to determine all the instructors on this committee, both as a point plus an arbitrary multiple of a vector and as a linear equation satisfied by the coordinates of any point (x, y) on the line.

   (i) Determine which committee satisfies the equation $[u]x + [u+1]y = [u]$.

Assignments for outline 8

1. Stephanie
2. Eugene
3. Tali
4. Sam
5. Vivian
6. Meagan
7. Christine
8. Mark K.
9. Mark Z.

Assignments for outline 9

1. Wei
2. Brian
3. Ed
4. Amie
5. Dennis
6. Zachary
7. Greg
8. Stephanie
9. Eugene
10. Tali
11. Sam
12. Vivian
13. Meagan
14. Christine
15. Mark K.
16. Mark Z.
17. Paul (unless someone wants to volunteer to share this with me)