MATHEMATICS S-152, SUMMER 2004
The MATHEMATICS OF SYMMETRY

Homework Assignment # 1
Due: Thursday, June 30, 2005

Reading

- Biggs, Section 27.3 (Symmetry)
- Biggs, Sections 5.2-5.4, 10.6, 12.5, 12.6, and 21.1 (Permutations)

Required Problems

1. Determine the number of symmetries of a regular n-gon. Specify how many are rotations about an axis perpendicular to the plane containing the n-gon and how many are reflections (or you can think of them as rotations) about an axis in the plane of the figure.

2. Build physical models of the five regular polyhedra, known as the Platonic solids. (Bring these to class on Thursday, July 1, and just include a note on your homework stating that you have done them).

3. Make a table listing the symmetries for each of the five regular polyhedra (tetrahedron, cube, octahedron, dodecahedron, icosahedron), and classify them by type. *(This is just a summary of results from the first class.)*

4. Make a table of compositions/products for the elements of the symmetry group of the square. *Hint*: Part of this exercise is to determine a good notation for the symmetries.

5. Let $\sigma = (1234)$ and $\tau = (123)(45)$ be two permutations of the set of 5 elements. The inverse of a permutation "undoes" the effect of that permutation so that $\sigma^{-1}\sigma$ is the identity. Compute σ^{-1}, τ^{-1}, $\sigma\tau$, and $\tau\sigma$, and write each of σ and τ as a product of transpositions. A *transposition* is a permutation that just interchanges two elements, like (12).

6. Let $\sigma = (1357)(246)$ be a permutation of the set of seven elements.
   (a) Find σ^2, σ^3, and σ^{-1}.
   (b) What is the smallest positive integer n such that σ^n is the identity? This n is defined to be the *order* of the element σ.

7. If you number the vertices of a tetrahedron as 1, 2, 3, 4, then any symmetry of the tetrahedron is a permutation of the set $\{1, 2, 3, 4\}$.
   (a) List the permutations that correspond to rotations through angles of 120 or 240 degrees.
   (b) List the permutations that correspond to rotations through an angle of 180 degrees.
   (c) Show that any these permutations can be written as the product of two transpositions.

Exploratory Problems

1. Find the coordinates in $\mathbb{R}^3$ for the vertices of the cube, octahedron, and tetrahedron in reasonable orientations.

2. Find the coordinates in $\mathbb{R}^2$ for the vertices of a square in a reasonable orientation. Then write down 2x2 matrices that represent a 90 degree counterclockwise rotation of the square, a reflection in a diagonal, and a reflection in a line parallel to two sides of the square.

3. Discuss the distinction between the symmetry groups of planar objects and those of solid objects. In particular, why are reflections considered valid symmetries of planar objects, but not of solid objects? (What *is* a reflection of a solid object?) How would the symmetry groups be different if we did/did not allow reflections?