MATHEMATICS S-152, SUMMER 2005
The MATHEMATICS OF SYMMETRY

Homework Assignment # 5
Due: Thursday, July 28, 2004

## Reading

- Read Biggs, Chapter 23, especially 23.1–23.4.
- Read the handout on Affine Geometry, up through the proof that addition is well defined.

## Required Problems

1. Consider the quotient ring $R = \mathbb{Z}_3[x]/\langle q(x) \rangle$, where $q(x) = x^3 + x + 1$. How many elements are in $R$? Compute the products $[2x+1][x^2]$ and $[x+2][x^2+2x+2]$ in $R$.

2. Exhibit an isomorphism between the non-zero elements of $\mathbb{F}_8$ (that is, the multiplicative group of the field) and the elements of $\mathbb{Z}_7$ (considered as an additive group).

3. Use Euclid's gcd algorithm to find the greatest common divisor of $x^3 + 4x^2 + 2x + 2$ and $2x^3 + 4x^2 + 3$ as polynomials in$\mathbb{Z}_5[x]$. (The gcd is a quadratic polynomial.)

4. The polynomial $q = x^2 + 3x + 3$ is irreducible over $\mathbb{Z}_5$ and so can be used to construct the field $\mathbb{F}_{25}$. Use Euclid's algorithm to find the inverse of $p = [x+2]$ in $\mathbb{F}_{25}$ by finding polynomials $m$ and $n$ such that $mp + nq = 1$.

5. Consider the quotient ring $R = \mathbb{Z}_5[x]/\langle q(x) \rangle$ where $q(x) = x^2 + 2$. How many elements are in $R$? Find the multiplicative inverse for the element $[ax+b]$, where $a \neq 0$. Find the orders of the elements $[x]$ and $[x+1]$.

6. Biggs, problem 3 on page 317.

## Exploratory Problems

9. Consider the ring $R = F[x]$, where $F$ is a field. Suppose $a(x), b(x) \in R$ are non-zero. Show that $deg(a(x)b(x)) = deg(a(x))deg(b(x))$. Construct a counterexample to show that this result is not always true in the ring $\mathbb{Z}_6[x]$.

10. Biggs, problem 5 on page 322.

11. (This is pasted in by hand because of the complicated diagram; so it will not appear in the Web version.)

12. (This is a second problem on the next page.)