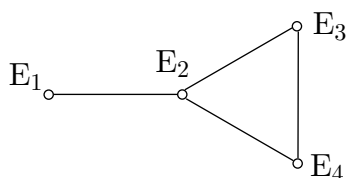


**MATH 191**  
**PRACTICE FINAL EXAM**

1. A standard deck of cards (52 cards; 13 each of spades, hearts diamonds and clubs) is shuffled and turned over one at a time. What are the odds that at every stage at least as many spades have been turned over as hearts?
2. A group of  $2n$  boys and  $2n$  girls is divided at random into two equal groups. What are the odds that each group will be equally divided into boys and girls? Give the exact answer, and then estimate this using Stirling's formula.
3. In a random walk along a line (at each epoch the particle moves 1 to the left or right with probability  $1/2$ ) starting at the origin, what are the odds that
  - (a) the particle returns to the origin at epoch  $2n$  ?
  - (b) the particle returns to the origin *for the first time* at epoch  $2n$  ?
4. A coin is tossed 5 times. What are the odds that there will be a streak of three heads or tails?
5. Seven balls are distributed randomly in 7 cells (so that there are  $7^7$  possible outcomes, all equally likely).
  - (a) What are the odds of some cell receiving four or more balls?
  - (b) What are the odds of some cell receiving four or more balls, given that exactly two of the cells are empty?
6. A book of 500 pages contains 250 misprints. Estimate the odds that a given page contains three or more misprints.
7. An ordinary (6-sided) die is rolled 6000 times. What are the odds of getting 1200 or fewer 6's?

8. Two ordinary dice are rolled. Let  $\mathbf{X}$  be the sum of the faces showing, and  $\mathbf{Y}$  the product.
- What are the expectations of  $X$  and  $Y$ ?
  - What are the variances of  $X$  and  $Y$ ?
  - What is the covariance  $\text{Cov}(\mathbf{X}, \mathbf{Y})$ ?
9. Here's a modification of the random walk on a graph discussed in the last homework: at each epoch the particle either stays put or moves to one of the adjacent nodes, all with equal probability (so that for example if there are  $\nu$  edges attached to a node, the odds that a particle at that node will stay put is  $\frac{1}{\nu+1}$ ).

Find the invariant distribution and mean recurrence times for this modified random walk on the graph below.



10. Let  $\mathbf{X}$  be the number of aces in a random bridge hand (that is, a random subset of 13 cards in a standard deck of 52).
- What are the expectation and variance of  $X$ ?
  - A paranoid bridge player keeps track, and finds that in 100 hands he has received 80 aces. What are the odds of this happening if the deals are truly random?