PROBLEM SET #3 - SOLUTIONS

11. If a family has $n \geq k$ children (otherwise the problem doesn't make much sense), then the probability that they have exactly k boys is $\binom{n}{k}2^{-n}$. The probability that a family has exactly n children is αp^n, so the probability of n children AND k boys is $\binom{n}{k}\left(\frac{p}{2}\right)^n$. If we sum this over all $n \geq k$, then we get the probability that a family has exactly k boys:

$$\sum_{n=k}^{\infty} \alpha \binom{n}{k} \left(\frac{p}{2}\right)^n = \alpha \frac{p^k}{2^k} \left(\sum_{n=0}^{\infty} \binom{n+k}{k} \left(\frac{p}{2}\right)^n \right).$$

I will leave out the inductive step that tells us that

$$\frac{1}{(1-x)^{k+1}} = \sum_{n=0}^{\infty} \binom{n+k}{k} x^n.$$

This follows immediately from Cauchy's product formula. This allows us to reduce the sum before to just

$$\frac{2\alpha p^k}{(2-p)^{k+1}}.$$

12. The probability of having more than 1 boy is the sum of the probabilities of have n boys for $n \geq 2$. This is just

$$\sum_{k=2}^{\infty} \frac{2\alpha p^k}{(2-p)^{k+1}} = \frac{p}{2-p} \sum_{k=1}^{\infty} \frac{2\alpha p^k}{(2-p)^{k+1}}.$$

The last sum is just the probability of having at least one boy, so the conditional probability is just $\frac{p}{2-p}$.