MATHEMATICS 191, FALL 2003
MATHEMATICAL PROBABILITY
Assignment #11

Problems to be handed in on Thursday, January 8:

These are fairly straightforward problems designed to encourage you to take a closer look at some of the results proved in lecture.

1. Problem 5.2.7 in G&S. Yes, this has a solution, but there are lots of details that need to be added. In the first part, give a concrete example. Write out the functions $G_1(s)$, $G_2(s)$, $G_3(s)$, and $G_4(s)$. Whether you use induction or repeated integration to do the second part, remember that $G_n(1) = 1$.

2. Apply Waring's Theorem to the simple problem where there are n independent events A_i, each of which occurs with probability p. In this case you already know the answer for $\mathbb{P}(X = i)$, but the challenge is to work out S_m and then manipulate binomial coefficients to get the answer by using Waring's Theorem.

3. Solve the "Bill Veeck midget" problem from assignment 7 by expanding the generating function for the "problem of the points,"

$$G(x, y) = \frac{y}{1-y}(1 - \frac{px}{1-qy})^{-1}$$

in a power series and keeping just the term in x^4y^3. Your result will match up term-for-term with one of the approaches from Assignment 7, problem 1 – which one?

4. According to the "hitting time theorem" (equivalent to the ballot theorem) the probability $f_1(n)$ of reaching level 1 for the first time after n steps of a random walk should equal $p_1(n)/n$, where $p_1(n)$ is the probability of being in level 1 after n steps. Confirm this by expanding the generating function $F_1(s)$ in a power series.

5. Use generating functions to prove that the probability that a symmetric random walk ($p = 0.5$) never goes to negative levels during the first $2n$ steps is equal to the probability of its being at level 0 after $2n$ steps. You can model your proof on the solution to problem 5.3.2b in G&S. As usual, they reverse the order of summation without comment. Please draw a diagram to show how you do it.