MATHEMATICS 191, FALL 2003
MATHEMATICAL PROBABILITY
Assignment #3

Problems to be discussed in section: All problems are from Grimmett and Stirzaker, 1000 Exercises in Probability. The solutions are all in the book!

Add together the number of letters in your first and last name. If the sum is odd, prepare problems 1,3, and 5. If it is even, prepare 2, 4, and 6.

1. Section 2.1, problem 4.
2. Section 2.7 , problem 3.
3. Section 2.2 , problem 1.
4. Section 2.7, problem 12.
5. Section 2.2, problem 3.
6. Section 2.5, problem 4.

Problems to be handed in on Thursday, October 9:

1. Prove Lemma 11 on p. 30 of G&S.
2. Using the same approach as in section 2.2, prove directly that

$$\mathbb{P}(\frac{1}{n}S_n \leq p - \epsilon) \leq e^{-\frac{1}{4}n\epsilon^2}$$

   for $\epsilon > 0$. You may adopt the same proof strategy as in the book and in lecture, but you may not assume the result that was proved. In other words, start with a lower limit of 0 and an upper limit less than n. Various signs will need to be changed from the original proof.

3. Suppose that the random variable X is uniformly distributed in the interval $(1, 27]$ and that the random variable Y is given by the function $Y = X^{\frac{1}{3}}$.
   (a) What is the probability of the event $2 < Y \leq 3$?
   (b) Determine the distribution function $F_Y(t)$ and a density function $f_Y(t)$ on the interval $1 < Y \leq 3$. Sketch graphs of both. Show that $F_Y(t)$ is consistent with your answer to part a.

Math 191 student Andrew Chi suggested in an email to me, "Perhaps with a few more restrictions, we would be able to prove a theorem that guarantees that the whole family is independent.

Do you think there might be something like that?"
Here is the best I could find (from Billingsley, *Probability and Measure*).

4. For events $A_1, ...A_n$, consider the 2^n equations

$$\mathbb{P}(B_1 \cap ... \cap B_n) = \mathbb{P}(B_1)...\mathbb{P}(B_n)$$

with $B_i = A_i$ or $B_i = A_i^c$ for each i.
Show that $A_1, ...A_n$ are independent if all these equations hold.

5. (a) Invent an example of identity (10) at the bottom of p. 29 in G&S based on three coin tosses, where A is "precisely 2 heads occur" and the events B_0, B_1, B_2 (lower index changed to 0 for convenience) refer to the number of heads in the last two tosses. One way to represent the indicator functions graphically was shown in class.

   (b) Prove the identity (10).