MATHEMATICS 191, FALL 2003
MATHEMATICAL PROBABILITY
Assignment #4

Problems that might be useful to look at before solving the problems to be handed in:
All problems are from Grimmett and Stirzaker, 1000 Exercises in Probability.
The solutions are all in the book!

Because next Monday is a legal holiday, it would be a violation of Massachusetts law for sections to meet!

1. Section 3.2, problem 3.
2. Section 3.2 , problem 4.
3. Section 3.4 , problem 9.
4. Section 3.6, problem 4.

Problems to be handed in on Thursday, October 16:

1. A baseball playoff series ends when either team wins 4 games. Team 1 has probability p of winning each game, independently of the other games. Let the random variable X be the number of games in the series. Calculate the probability mass function for X for the case $p = 0.6$, and determine its expectation and variance.

2. In the final game of the World Series, a baseball manager has n pitchers available in his bullpen. Of these, r will throw nothing but strikes, while the other $n - r$ will throw nothing but balls. Nobody knows which is which, of course! The manager brings pitchers into the game in a random order. The random variable X is the number of pitchers he must try before finding one who throws strikes. Find the probability mass function for X, and calculate its expectation. Assume $r > 0$, though in real life this is not necessarily so.

3. Contestants A, B, and C are participating in a home-run hitting contest. They go in the order ABCABCA.... Each participant has probability $p = \frac{1}{3}$ of hitting a home run on each attempt.

   (a) What is the probability that both A and B hit home runs before C does?

   (b) What is the probability that the third home run of the contest, but neither the first nor the second, is hit by C?

4. n "Iraqi most wanted" prisoners are languishing in Guantanamo. They persuade a judge that depriving them of playing cards is a human-rights violation, and so they are given the n U.S. military cards that have their pictures. Let h_n be the number of ways of dealing these cards, one to each prisoner, so that no prisoner gets his own card.

   Now one more prisoner arrives, then another, and their cards are added to the deck. Analyze the situation to find a formula for h_{n+2} in terms of h_{n+1} and h_n. By solving this recurrence relation, get an alternative derivation of the solution to the matching problem.

5. Invent a baseball version of Simpson's paradox, where

   - Event A is "the outcome is a hit".
   - Event B is "the hitter is player 1, not player 2'".
   - Event C is "there were runners in scoring position."

   The "paradox" should be that player 1 has a higher batting average than player 2 both with and without runners in scoring position, but that player 2 has a higher overall batting average. You might make one of the players be a pinch hitter who is used mostly with runners in scoring position, while the other is a leadoff man who often bats with the bases empty. Feel free to make the overall probability of a hit quite different with and without runners in scoring position.

   After you have formulated your "paradox" as a set of eight probabilities in a table, show a calculation of $\mathbb{P}(A|B)$ in terms of $\mathbb{P}(A|B \cap C)$ and $\mathbb{P}(A|B \cap C^c)$ that makes it clear how you rigged conditional probabilities like $\mathbb{P}(C|B)$ in setting up the "paradox".