

Problem from p. 49 of Stirzaker, Elementary Probability

20. The function  $d(x, y)$  is defined on the event space by  $d(A, B) = \mathbb{P}(A \Delta B)$ .

(a) Show that for any events  $A, B, C$

$$d(A, B) + d(B, C) - d(A, C) = 2\mathbb{P}(A \cap B^c \cap C) + \mathbb{P}(A^c \cap B \cap C^c)$$

(b) When is  $d(A, B)$  zero?

(c) Let  $A_1, A_2, \dots$  be a monotone sequence of events such that  $A_i \subseteq A_j$  for  $i \leq j$ . Show that for  $i \leq j \leq k$ ,

$$d(A_i, A_k) = d(A_i, A_j) + d(A_j, A_k)$$