Math 192r, Problem Set #5
(due 10/4/01)

1. There is a unique polynomial of degree d such that $f(k) = 2^k$ for $k = 0, 1, ..., d$. What is $f(d+1)$? What is $f(-1)$?

2. One basis for the space of polynomials of degree less than d is the monomial basis $1, t, t^2, ..., t^{d-1}$. Another is the shifted monomial basis $1, (t+1), (t+1)^2, ..., (t+1)^{d-1}$. Call these bases $u_1, ..., u_d$ and $v_1, ..., v_d$ respectively.

   (a) Derive a formula for the entries of the change-of-basis matrix M expressing the u_i's as linear combinations of the v_j's.

   (b) Derive a formula for the entries of the change-of-basis matrix N expressing the v_j's as linear combinations of the u_i's.

   (c) From the description of M and N as basis-change matrices, we know that $MN = NM = I$. Forgetting for the moment what M and N mean, rewrite the assertions $MN = NM = I$ as binomial coefficient identities, and prove them either algebraically or bijectively.