Math 19 Homework

1. a) $h(0)=0,\ h(R)=0$

$\underline{c<0}$ $h(0)=\alpha=0,\ h(R)=\beta\sin(\sqrt{-c}R)=0$

$\Rightarrow \beta=0$ or $\sin(\sqrt{-c}R)=0$

$\Rightarrow \sqrt{-c}\cdot R=n\pi$ (n integer

$\Rightarrow c=-\frac{n^2\pi^2}{R^2}$ $\boxed{c=-\frac{n^2\pi^2}{R^2},\ \beta\neq 0, \alpha=0}$

$\underline{c=0}$ $h(0)=\alpha=0,\ h(R)=\beta R=0 \Rightarrow \boxed{\alpha=\beta=0}$

$\underline{c>0}$ $h(0)=\alpha+\beta=0 \Rightarrow \beta=-\alpha$

$h(R)=\alpha e^{\sqrt{c}R}-\alpha e^{-\sqrt{c}R}=0 \Rightarrow \boxed{\alpha=\beta=0}$

c) $\left(\frac{dh}{dx}\right)(0)=0,\ h(R)=0$

$\underline{c<0}$ $h(R)=\alpha\cos(\sqrt{-c}R)+\beta\sin(\sqrt{-c}R)=0$

$\left.\frac{dh}{dx}\right|_{x=0}=-\alpha\sqrt{-c}\ \sin(\sqrt{-c}\cdot 0)+\beta\sqrt{-c}\cdot\cos(\sqrt{-c}\cdot 0)=\beta\sqrt{-c}=0 \Rightarrow$

$\beta=0$

so $\alpha\cos(\sqrt{-c}R)=0 \Rightarrow \cos(\sqrt{-c}R)=0 \Rightarrow \sqrt{-c}R=\pi(n/2) \Rightarrow$

$\boxed{c=-\frac{n^2\pi^2}{4R^2},\ \alpha\neq 0, \beta=0}$

$\underline{c=0}$, $\underline{c>0}$ trivial solutions

e) $h(0)=0,\ h(R)=1$

$\underline{c>0}$ $h(0)=\alpha+\beta=0 \Rightarrow \beta=-\alpha$

$h(R)=\alpha e^{\sqrt{c}R}+\beta e^{-\sqrt{c}R} \Rightarrow \boxed{\alpha=\frac{1}{e^{\sqrt{c}R}-e^{-\sqrt{c}R}},\ \beta=-\alpha}$

$\underline{c=0}$ $\boxed{\alpha=0,\ \beta=1/R}$

$\underline{c<0}$ $\boxed{\alpha=0,\ \beta=\frac{1}{\sin(\sqrt{-c}R)}}$

1. F) $h(0) = -1$, $h(R) = 1$

$\underline{c > 0}$ $\boxed{\beta = \dfrac{1 + e^{\sqrt{c}R}}{e^{-\sqrt{c}R} - e^{\sqrt{c}R}}}$ $\boxed{\alpha = -1 - \dfrac{1 + e^{\sqrt{c}R}}{e^{-\sqrt{c}R} - e^{\sqrt{c}R}}}$

$\alpha = -1 - \beta$

$\underline{c = 0}$ $\boxed{\alpha = -1, \ \beta = \dfrac{2}{R}}$

$\underline{c < 0}$ $\boxed{\alpha = -1, \ \beta = \dfrac{1 + \cos(\sqrt{-c}R)}{\sin(\sqrt{-c}R)}}$

2. Find $u(t,x)$ that is not 0 everywhere and obeys

$\frac{\partial}{\partial t} u(t,x) = 2 \frac{\partial^2}{\partial x^2} u(t,x)$ when $0 \le x \le 10$ and obeys (see p. 264 for equations w/ α, β)

a) $\left(\frac{d}{dx} u\right)(t,0) = 0$ and $\left(\frac{d}{dx} u\right)(t,10) = 0$ for all t

$\underline{\lambda = 0}$ $\frac{dB}{dx} = \alpha = 0$ so $u(t,x) = A(0)e^{\lambda t} \cdot \beta$, β a constant

b) $\left(\frac{d}{dx} u\right)(t,0) = 0$ and $u(t,10) = 0$

$\underline{\lambda = 0}$ $\frac{dB}{dx} = \alpha = 0$, $\beta = 0$

$\underline{\lambda > 0}$ $\alpha = \beta = 0$

$\underline{\lambda < 0}$ $u(t,x) = A_0 e^{\lambda t} \cdot \alpha \cos\left(\frac{\pi}{20} x\right)$

c) $u(t,0) = 0$ and $\left(\frac{d}{dx} u\right)(t,10) = 0$

$\lambda > 0$ $\alpha = \beta = 0$

$\lambda = 0$ $\alpha = \beta = 0$

$\lambda < 0$ $u(t,x) = A_0 e^{\lambda t} \cdot \beta \sin\left(\frac{\pi x}{20}\right)$

3. $\frac{du}{dt} = -c\frac{\partial u}{\partial x} + ru$

$u(t,x) = A(t)B(x)$ $\qquad \frac{dA}{dt} = \lambda A$

$A(t) = A(0)e^{\lambda t}$ $\qquad \frac{dB}{dx} = \frac{r-\lambda}{c} \cdot B = K \cdot B$

$\frac{dB}{dx} = KB \Rightarrow B(x) = B(0)e^{Kx}$

$u(t,x) = A(0)e^{\lambda t} \cdot B(0)e^{Kx}$, where $K = \frac{r-\lambda}{c}$