HW 7 Key Math 19

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1: $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - x^2 - 2xy \\ 2y - y^2 - 3xy \end{pmatrix}$ $D = \begin{pmatrix} 1-2x-2y & -2x \\ -3y & 2-2y-3x \end{pmatrix}$

$D(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ det $D = 2 > 0$, tr $D = 3 > 0$ unstable

$D(1,0) = \begin{pmatrix} -1 & -2 \\ 0 & -1 \end{pmatrix}$ det $D = 1 > 0$, tr $D = -2 < 0$ stable * the book is wrong here

$D(0,2) = \begin{pmatrix} -3 & 0 \\ -6 & -2 \end{pmatrix}$ det $D = 6 > 0$, tr $D = -5 < 0$ stable

$D(.6, .2) = \begin{pmatrix} -.6 & -1.2 \\ -.6 & -.2 \end{pmatrix}$ det $D = -.6$, tr $D = -.8$ unstable

3(a) $h = x^2y^3$ $\frac{\partial h}{\partial x} = 2xy^3$ $\frac{\partial h}{\partial y} = 3y^2x^2$

$\frac{\partial^2 h}{\partial x^2} = 2y^3$ $\frac{\partial^2 h}{\partial x \partial y} = 6xy^2$

$\frac{\partial^2 h}{\partial y^2} = 6x^2y$ $\frac{\partial^2 h}{\partial y \partial x} = 6xy^2$

(b) $h = x\cos(xy)$ $\frac{\partial h}{\partial x} = \cos(xy) - xy\sin(xy)$ $\frac{\partial h}{\partial y} = -x^2\sin(xy)$

$\frac{\partial^2 h}{\partial x^2} = -2y\sin(xy) - xy^2\cos(xy)$ $\frac{\partial^2 h}{\partial x \partial y} = -2x\sin(xy) - x^2y\cos(xy)$

$\frac{\partial^2 h}{\partial y^2} = -x^3\cos(xy)$ $\frac{\partial^2 h}{\partial y \partial x} = -2x\sin(xy) - x^2y\cos(xy)$

(c) $h = \sin(x+y^2)$ $\frac{\partial h}{\partial x} = \cos(x+y^2)$ $\frac{\partial h}{\partial y} = 2y\cos(x+y^2)$

$\frac{\partial^2 h}{\partial x^2} = -\sin(x+y^2)$ $\frac{\partial h}{\partial x \partial y} = -2y\sin(x+y^2)$

$\frac{\partial^2 h}{\partial y^2} = 2\cos(x+y^2) - 2y\sin(x+y^2)\cdot 2y$ $\frac{\partial h}{\partial y \partial x} = -2y\sin(x+y^2)$

(d) $h = xe^y$ $\frac{\partial h}{\partial x} = e^y$ $\frac{\partial h}{\partial y} = xe^y$

$\frac{\partial h}{\partial x^2} = 0$ $\frac{\partial h}{\partial x \partial y} = e^y$

$\frac{\partial h}{\partial y^2} = xe^y$ $\frac{\partial h}{\partial y \partial x} = e^y$

in all cases $\frac{\partial h}{\partial y \partial x} = \frac{\partial h}{\partial x \partial y}$

4(a) $\int_0^1\int_{-1}^2 (1)\,dy\,dx = \int_0^1 \left(y\Big]_{-1}^{2} \right) dx = \int_0^1 3\,dx = 3$

(b) $\int_0^1\int_{-1}^1 x\,dx\,dy = \int_0^1 \left(\frac{x^2}{2}\Big]_{-1}^{1} \right) dy = \int_0^1 \left(\frac{1}{2} - \frac{1}{2} \right) dy = 0$

(c) $\int_0^1\int_0^1 (x+y)\,dy\,dx = \int_0^1 \left(xy + \frac{y^2}{2}\Big]_0^1 \right) dx = \int_0^1 \left(x + \frac{1}{2} \right) dx = \left(\frac{x^2}{2} + \frac{x}{2} \right)\Big]_0^1 =$

$1/2 + 1/2 = 1$

(d) $\int_{-2}^1\int_2^3 (xy)\,dy\,dx = \int_{-2}^1 \left(\frac{xy^2}{2}\Big]_2^3 \right) dx = \int_{-2}^1 \left(\frac{9x}{2} - \frac{4x}{2} \right) dx = \frac{5x^2}{4}\Big]_{-2}^{1} =$

$\frac{5}{4} - 5 = \frac{-15}{4}$

(e) $\int_0^1\int_0^1 \cos(xy)\,dy\,dx = \int_0^1 \left(\frac{\sin(xy)}{x}\Big]_0^1 \right) dx = \int_0^1 \frac{1}{x}\sin(x)\,dx \approx .946$

p. 177 1. (a). $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1+2 \\ 0+1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

(b). $\begin{pmatrix} 3 & 2 \\ -5 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3+6 \\ -5+12 \end{pmatrix} = \begin{pmatrix} 9 \\ 7 \end{pmatrix}$

(c). $\begin{pmatrix} .5 & .2 \\ 8 & .4 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0+.2 \\ 0+.4 \end{pmatrix} = \begin{pmatrix} .2 \\ .4 \end{pmatrix}$

2. (a) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix} = \begin{pmatrix} 2+2 & 5+12 \\ 0+1 & 0+6 \end{pmatrix} = \begin{pmatrix} 4 & 17 \\ 1 & 6 \end{pmatrix}$

$\begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2+0 & 4+5 \\ 1+0 & 2+6 \end{pmatrix} = \begin{pmatrix} 2 & 9 \\ 1 & 8 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 9 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0+9 & 0+9 \\ 6-4 & 15-4 \end{pmatrix} = \begin{pmatrix} 9 & 9 \\ 2 & 11 \end{pmatrix}$

$\begin{pmatrix} 2 & 5 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 9 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} 0+15 & 18-20 \\ 0+3 & 9-4 \end{pmatrix} = \begin{pmatrix} 15 & -2 \\ 3 & 5 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix}\begin{pmatrix} 0 & 9 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} 0+15 & 18-20 \\ 0+18 & 9-24 \end{pmatrix} = \begin{pmatrix} 15 & -2 \\ 18 & -15 \end{pmatrix}$

$\begin{pmatrix} 0 & 9 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix} = \begin{pmatrix} 0+9 & 0+56 \\ 6-4 & 15-24 \end{pmatrix} = \begin{pmatrix} 9 & 56 \\ 2 & -9 \end{pmatrix}$

3.(a) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ det = 1 tr = 2

(b) $\begin{pmatrix} 0 & 9 \\ 3 & -4 \end{pmatrix}$ det = -27 tr = -4

$\begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix}$ det = 7 tr = 8

4. either:

$$M \cdot X = \lambda X$$

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1+0 \\ 0+0 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\therefore$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is an eigenvector for the eigen value $\lambda = 1$

or: find eigenvalues of $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and solve for eigenvectors, showing that $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ meets these criteria.