Midterm # 2 Ch. 13-23

May bring a 4x6 notecard / half page of notes
only through Ch. 23, Not 24

Advection Equation

$\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x} + f(u)$

Advection Models - be able to identify (like homework question) which model Advection, Diffusion, LaPlace fits situation - just answer which one fits

Diffusion Equation

$\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial x^2} + f(u)$

LaPlace's Equation

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ steady state - time independent in 2-D

No reaction term, b/c nothing reacts

Separation of variables

ex) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $\quad u(t,x) = A(t)B(x) \Rightarrow A'B = AB'' \Rightarrow \frac{A'}{A} = \frac{B''}{B} = \lambda$

$A' = \lambda A$ $\quad A(t) = A_0 e^{\lambda t}$

$B'' = \lambda B$

3 cases

$$B(x) = \begin{cases} \alpha e^{\sqrt{\lambda}x} + \beta e^{-\sqrt{\lambda}x} & \lambda > 0 \\ = \alpha + \beta x & \lambda = 0 \\ = \alpha\sin(\sqrt{-\lambda}\,x) + \beta\cos(\sqrt{-\lambda}\,x) & \lambda < 0 \end{cases}$$

Now: Apply Boundary conditions to find α + β

say we have: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 7u$

$$A'B = AB'' + 7AB$$

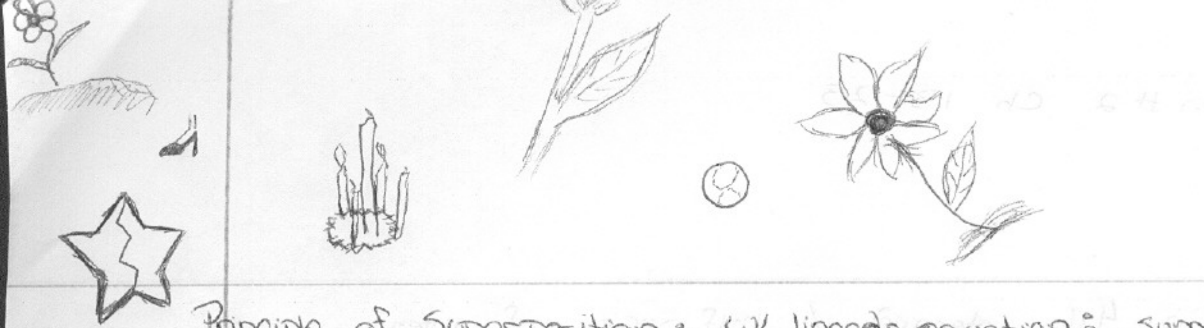
$$A'/A = B''/B + 7 = \lambda$$

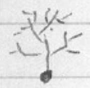
$u(0,x)$ = initial condition ; $u(t,0)$ = boundary condition

or $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$ $\quad u = A(x)B(y)$

$$A''B = AB''$$

$\frac{A''}{A} = B''/B = \lambda$ (Now need to do 3 cases for **BOTH** $A(x)$ + $B(y)$).





Principle of Superposition: w/ linear equation: sum of solutions are solutions; multiples of solutions are solutions

Stability Criteria:

$U_e = U_e(x)$ No pair $[\lambda, g(x)]$ s.t. $\lambda \geq 0$

$g \neq 0$ & $\lambda g = \frac{d^2 g}{dx^2} + z(x) g$

$z(x) = \frac{\partial f}{\partial u}(U_e)$

$g'(0) = g'(L) = 0$ $\boxed{\text{or}}$ $g(0) = g(L) = 0$

suppose $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u(u-5)$ $U_e = 5$ or $U = 0$

so $f'(u) = 2u - 5$ b/c $f(u) = u^2 - 5u$

$F(u_e) = 2(5) - 5 = 5 = z(x)$

Maximum Principle: $\lambda g = \frac{\partial^2 g}{\partial x^2} - (x^2+1) g$; $g(0) = g(1) = 0$

if g has max $\oplus = \ominus$ $\ominus \oplus$ $\therefore \lambda < 0$ b/c RHS < 0

if g has min $\ominus = \oplus$ $\ominus \ominus$ $\therefore \lambda < 0$ b/c RHS > 0

Traveling Waves Basics: (probably not much computation)

Poincaré - Bendixson Theorem

basin of attraction w/ 1 repelling eq. pt = periodic solutions } check boundaries & eq. pt.

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f(u)$ $f(u) = ru(1-u)$

$u(t,x) = f(x - ct)$; $0 \le f \le 1$; $\to 0$

$= f(\xi)$ $\xi \to -\infty$ $f \to 1$; $\xi \to \infty$ $f \to 0$

Know what to do with Null clines!

system $\frac{\partial x}{\partial t} = f(x,y)$ $\quad H = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix}$ & sub in eq point

$\frac{\partial y}{\partial t} = g(x,y)$

LaPlace is a steady-state

if you have something like $\frac{\partial x^2}{\partial t^2} + 5x\frac{\partial x}{\partial t} - \cos x = 0$

let $y = \frac{dx}{\partial t}$ $\Rightarrow$ $\frac{dy}{\partial t} + 5xy - \cos x = 0$

Now you have a system of first order eq. & can use Null clines & phase plane

how to go from $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + ru(1-u)$ p. 353

to $-c\frac{\partial f}{\partial s} = \frac{\partial^2 f}{\partial s^2} + rf(1-f)$

let $u(t,x) = f(x-ct) = f(s)$ so $s = x-ct$

$\frac{\partial u}{\partial t} = \frac{\partial f}{\partial t} = \frac{\partial f}{\partial s}\cdot\frac{\partial s}{\partial t} = -c\frac{\partial f}{\partial s}$ b/c $\frac{\partial s}{\partial t} = -c$ (This is chain rule!)

More examples of this in homework solution.

ex. # 3 pg. 489

$\frac{\partial u}{\partial t} = -2\frac{\partial u}{\partial x}$ & $u(t,1) = \frac{1}{1+e^t}$

so: $u = e^{-rt} f(x-ct)$ but $r = 0$

$= f(x-ct)$ $c = 2$

so $u = f(x-2t)$

$u(t,1) = f(1-2t) = \frac{1}{1+e^t} =$

$1-2t = s$

$t = \frac{s-1}{-2} = \frac{1-s}{2}$

so $f(s) = \frac{1}{1+e^{(1-s)/2}} \Rightarrow \left(1+e^{(1-x+2t)/2}\right)^{-1} = f(x-2t)$

so: $u = \left(1+e^{(1-x+2t)/2}\right)^{-1}$