

Math 19. Lecture 11

Nonlinear Stability Revisited

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1 Stability Criteria for a One-Component System

An equilibrium solution for

$$\frac{dx}{dt}(t) = f(x)$$

is a constant solution $x(t) = x_0$ such that

$$\frac{dx}{dt} < 0$$

where $x(t) > x_0$ and $x(t)$ is close to x_0 and

$$\frac{dx}{dt} < 0$$

where $x(t) < x_0$ and $x(t)$ is close to x_0 . For example,

$$\frac{dx}{dt} = 9x - x^2 - 18 = (x - 3)(6 - x)$$

has equilibrium points at $x = 6$ and $x = 3$. The solution $x = 6$ is stable while the solutions $x = 3$ is unstable.

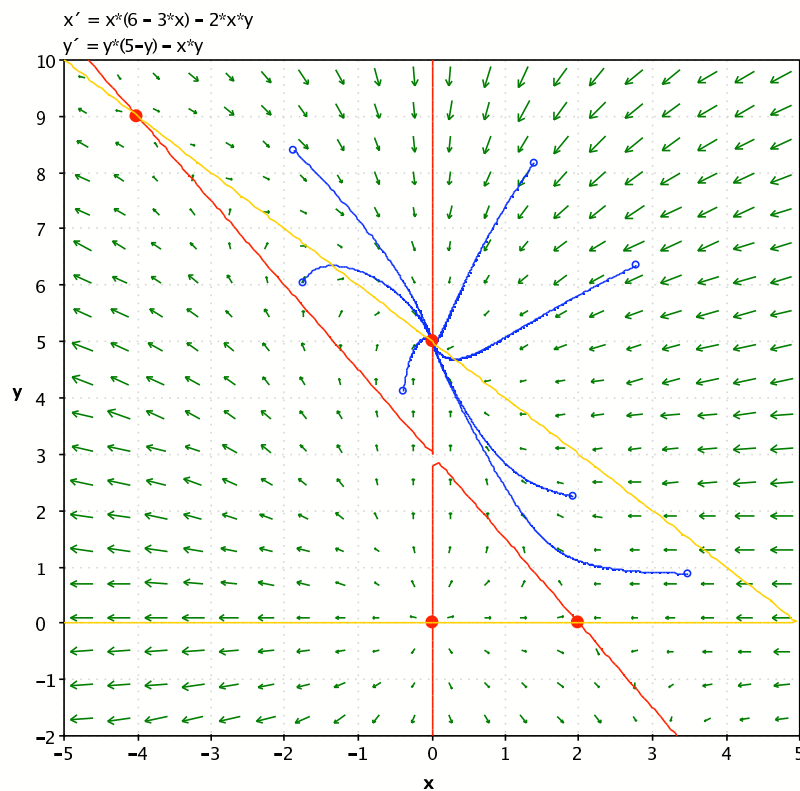
2 Stability Criteria for a Two-Component System

Recall the competition of species model. If two species compete for the same resources, then we can model this situation as

$$\begin{aligned}x' &= x(a_1 - b_1x) - \alpha_1xy \\y' &= y(a_2 - b_2y) - \alpha_2xy.\end{aligned}$$

For example, consider the system

$$\begin{aligned}x' &= x(6 - 3x) - 2xy \\y' &= y(5 - y) - xy.\end{aligned}$$



The x nullclines are $x = 0$ and $y = -\frac{3}{2}x + 3$. The y nullclines are $y = 0$ and $y = -x + 5$. The equilibrium points for this system are

$$\begin{pmatrix} -4 \\ 9 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \end{pmatrix},$$

Since

$$D = \begin{pmatrix} 6 - 6x - 2y & 2x \\ -y & 5 - 2y - x. \end{pmatrix}$$

Since

$$D(-4, 9) = \begin{pmatrix} 12 & -8 \\ -9 & -9 \end{pmatrix}$$

$$D(0, 5) = \begin{pmatrix} -4 & 0 \\ -5 & -5 \end{pmatrix}$$

$$D(0, 0) = \begin{pmatrix} 6 & 0 \\ 0 & 5 \end{pmatrix}$$

$$D(2, 0) = \begin{pmatrix} -6 & 0 \\ 0 & 3 \end{pmatrix},$$

$(0, 5)$ is the only stable equilibrium point.

3 Multiple Integration

Recall single variable integration. For example,

$$\int_0^1 x^2 dx = \frac{x^3}{3} \Big|_0^1 = \frac{1}{3}.$$

In two variables, let $R = [0, 2] \times [0, 1]$. Then

$$\iint_R x^2 y dA = \int_0^2 \int_0^1 x^2 y dy dx = \int_0^1 \int_0^2 x^2 y dx dy$$

Homework

- Chapter 10. Exercises 1, 3, 4; p. 172.

Readings and References

- C. Taubes. *Modeling Differential Equations in Biology*. Prentice Hall, Upper Saddle River, NJ, 2001. Chapter 10.

- “Hopes for the Future: Restoration Ecology and Conservation Biology,” pp. 153–166.