Math 19 Problem Set #19: p410-411 Ex. 1ac, 2ac

1. For $x(t)$, $\frac{dx}{dt} = f(x) + c$, find c where # eq. pts. changes from 3 to 1.

(a) $f(x) = x^3 - 3x$

$\frac{df}{dx} = 3x^2 - 3 = 0 \qquad 3(x^2-1) = 3(x+1)(x-1) = 0 \qquad x=1,\ x=-1$

eq. pts. change when $x(t)$ has a local max/min on x-axis.

local max/min occur when $\frac{df}{dx}(x_0) = 0$. then $c = -f(x_0)$

$f(1) = 1-3 = -2 \qquad \boxed{c=2} \qquad f(-1) = -1+3 = 2 \qquad \boxed{c=-2}$

(c) $f(x) = 2x^3 + 9x^2 + 12x + 7$

$\frac{df}{dx} = 6x^2 + 18x + 12 = 6(x^2+3x+2) = 6(x+2)(x+1) = 0 \qquad x=-2,\ x=-1$

$f(-2) = -16 + 36 - 24 + 7 = 3 \qquad \boxed{c=-3}$

$f(-1) = -2 + 9 - 12 + 7 = 2 \qquad \boxed{c=-2}$

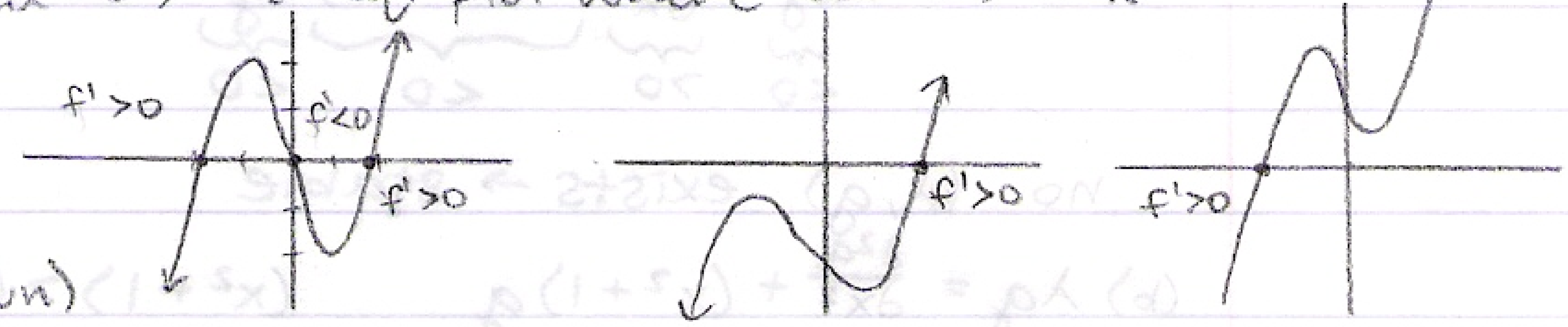
2. Part 1: Verify the following: Given a function $f(x,y)$, y fixed, such that $\lim_{x\to\infty} f(x,y) = \infty$ & $\lim_{x\to-\infty} f(x,y) = -\infty$,

(# eq. pts. where $\frac{df}{dx} > 0$) − (# eq. pts. where $\frac{df}{dx} < 0$) = 1.

(a) $f(x,y) = x^3 - 3x + y$

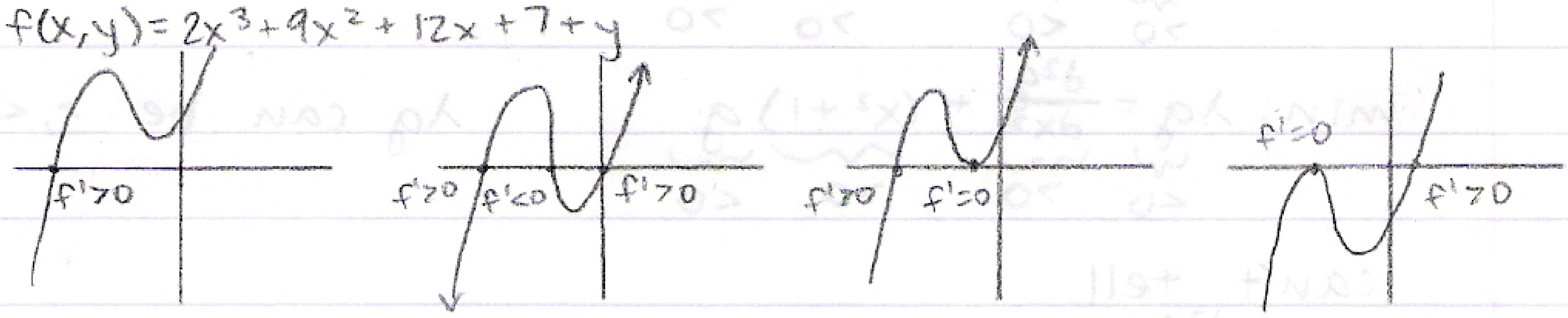
consider all y

(only 3 cases shown)

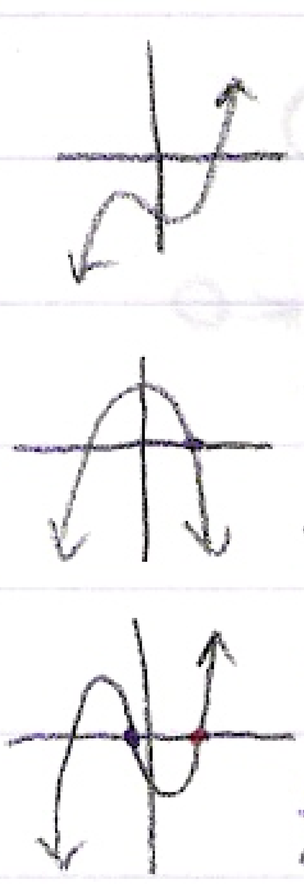


$2-1=1$ ✓ $\qquad 1-0=1$ ✓ $\qquad 1-0=1$ ✓

(c) $f(x,y) = 2x^3 + 9x^2 + 12x + 7 + y$



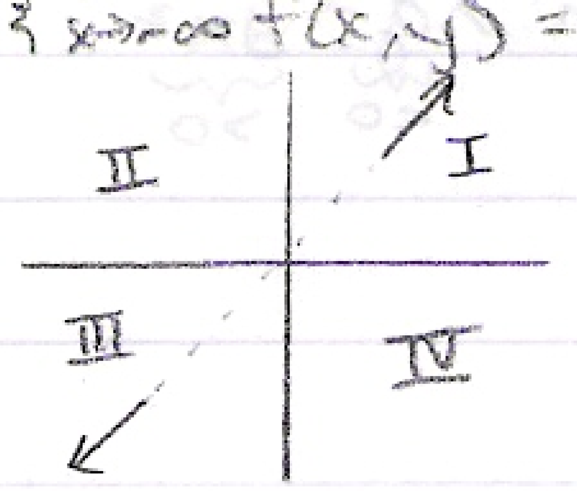
$1-0=1$ ✓ $\qquad 2-1=1$ ✓ $\qquad 1-0=1$ ✓ $\qquad 1-0=1$ ✓



$\frac{df}{dx} > 0: 1$
$\frac{df}{dx} < 0: 0$ ✓
$\frac{df}{dx} > 0: 1$
$\frac{df}{dx} < 0: 0+1$ ✗
$\frac{df}{dx} > 0: 1+1$
$\frac{df}{dx} < 0: 0+1$ ✓

Part 2: Justify the previous assertion or prove it false.

The assertion is true. For the specified functions ($\lim_{x\to\infty} f(x,y) = \infty$ & $\lim_{x\to-\infty} f(x,y) = -\infty$), f must be increasing in quadrant III & I,



so there must be a net increase over the x-axis of 1. In other words, each time $\frac{df}{dx} < 0$ when f crosses the x-axis, f must also increase over the x-axis an additional time to satisfy the conditions.