HW #5: Chapter 14.
Ex: 1, 3, 5, 6, 8, 10(a,c,e), 12 (a,c,e)
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Ex 1.

$f(t,x) = \sin(x-3t)$
$\frac{\partial f}{\partial t} = -3\cos(x-3t)$
$\frac{\partial f}{\partial x} = \cos(x-3t)$

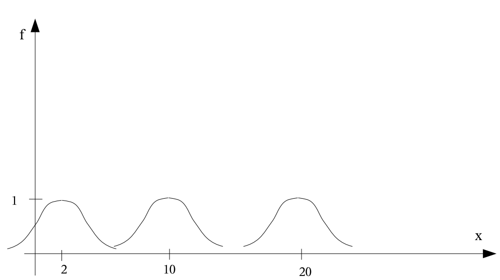
Ex. 2

$f(t,x) = t^{-1/2}e^{-x^2/t}$
$\frac{\partial f}{\partial t} = -\frac{1}{2}t^{-\frac{3}{2}}e^{-\frac{x^2}{t}} + \frac{x^2}{t^2}e^{-\frac{x^2}{t}}t^{-\frac{1}{2}} = -\frac{1}{2}t^{-\frac{3}{2}}e^{-\frac{x^2}{t}} + \frac{x^2}{t^{5/2}}e^{-\frac{x^2}{t}} = e^{-\frac{x^2}{t}}(-\frac{1}{2}t^{-3/2} + \frac{x^2}{t^{5/2}})$
$\frac{\partial f}{\partial x} = t^{-\frac{1}{2}}e^{-\frac{x^2}{t}}(-\frac{2x}{t}) = -2t^{-3/2}xe^{-\frac{x^2}{t}}$

Ex. 5

Sketch the graph for $f(1,x)$, $f(5,x)$, $f(10,x)$ on the same piece of graph paper (same plane of coordinates).

$f(t,x) = e^{-(x-2t)^2}$
$f(1,x) = e^{-(x-2)^2}$
$f(5,x) = e^{-(x-10)^2}$
$f(10,x) = e^{-(x-20)^2}$



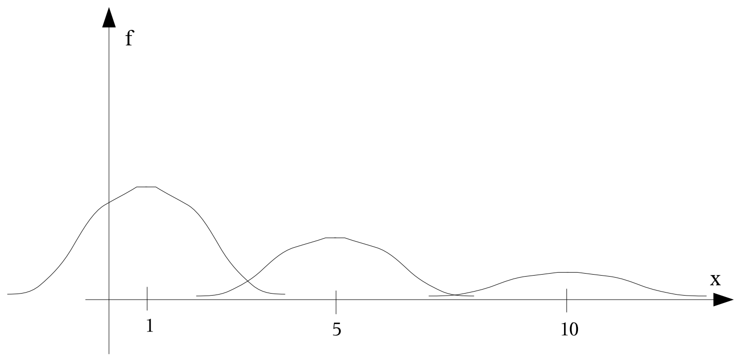
Ex. 5

$f(t,x) = e^{-t}(1+(t-x)^2)^{-1}$

$f(1,x) = e^{-1}(1+(1-x)^2)^{-1}$

$f(5,x) = e^{-5}(1+(5-x)^2)^{-1}$

$f(10,x) = e^{-10}(1+(10-x)^2)^{-1}$



Ex. 8

Solution for equation 4: $u(t,x) = e^{-\tau t} f(x-ct)$.

In ex. #5, $f(t,x) = e^{-(x-2t)^2} \Rightarrow f(x-ct) = e^{-(x-2t)^2}$. Therefore, $c = 2$.

$e^{-(x-2t)^2} = e^{-\tau t} f(x-ct) \Rightarrow e^{-\tau t} = 1 \Rightarrow \tau t = 0$. Thus, $\tau = 0$.

In ex. #4, $f(t,x) = e^{-t}(1-(t-x)^2)^{-1}$.

$f(x-ct) = (1-(t-x)^2)^{-1} = (1-(x-t)^2)^{-1} \Rightarrow c = 1$.

$e^{-\tau t} f(x-ct) = e^{-t}(1-(x-t)^2)^{-1} \Rightarrow \tau = 1$.

Solution for equatio 8: $u(t,x) = \frac{a}{t^{\frac{1}{2}}} e^{-x^2/4\mu t}$.

In ex. #7, $f(x,t) = t^{-\frac{1}{2}} e^{-x^2/t}$

$e^{-x^2/t} = a e^{-x^2/4\mu t} \Rightarrow -\frac{x^2}{t} = -\frac{x^2}{4\mu t} \Rightarrow 1 = \frac{1}{4\mu} \Rightarrow \mu = \frac{1}{4}$.

Ex. 10

Verify that the following are solutions to equation 8: $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2}$.

a) $u(t,x) = e^{\lambda t} e^{x\left(\frac{\lambda}{\mu}\right)^{\frac{1}{2}}}$

$\frac{\partial u}{\partial t} = \lambda e^{\lambda t} e^{x\left(\frac{\lambda}{\mu}\right)^{\frac{1}{2}}}$

$\frac{\partial u}{\partial x} = e^{\lambda t} \sqrt{\frac{\lambda}{\mu}} e^{x\left(\frac{\lambda}{\mu}\right)^{\frac{1}{2}}}$

$\frac{\partial^2 u}{\partial x^2} = \frac{\lambda}{\mu} e^{\lambda t} e^{x\left(\frac{\lambda}{\mu}\right)^{\frac{1}{2}}}$

Thus, $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2}$.

c) $u(t,x) = a + bx$, where a, b are constants.

$\frac{\partial u}{\partial t} = 0$

$\frac{\partial u}{\partial x} = b$

$\frac{\partial^2 u}{\partial x^2} = 0$

Thus, $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2}$.

d) $u(t,x) = e^{-\lambda t} \sin\left(x \left(\frac{\lambda}{\mu}\right)^{\frac{1}{2}}\right)$.

$\frac{\partial u}{\partial t} = -\lambda e^{-\lambda t} \sin\left(x \left(\frac{\lambda}{\mu}\right)^{\frac{1}{2}}\right)$

$\frac{\partial u}{\partial x} = e^{-\lambda t} \sqrt{\frac{\lambda}{\mu}} \cos\left(x \left(\frac{\lambda}{\mu}\right)^{\frac{1}{2}}\right)$

$\frac{\partial^2 u}{\partial x^2} = -\frac{\lambda}{\mu} e^{-\lambda t} \sin\left(x \left(\frac{\lambda}{\mu}\right)^{\frac{1}{2}}\right)$

Thus, $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2}$.

Ex. 12

Verify that the following are solutions to equation 9: $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2} + \tau u$.

a) $u(t,x) = e^{\lambda t} e^{x\left(\frac{\lambda - \tau}{\mu}\right)^{\frac{1}{2}}}$.

$\frac{\partial u}{\partial t} = \lambda e^{\lambda t} e^{x\left(\frac{\lambda - \tau}{\mu}\right)^{\frac{1}{2}}} = \lambda u(t,x)$

$\frac{\partial u}{\partial x} = \sqrt{\frac{\lambda - \tau}{\mu}} u(t,x)$

$\frac{\partial^2 u}{\partial x^2} = \frac{\lambda - \tau}{\mu} u(t,x)$

Thus, $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2} + \tau u$.

c) $u(t,x) = e^{\tau t}(a + bx)$.

$\frac{\partial u}{\partial t} = \tau e^{\tau t}(a + bx) = \tau u(t,x)$

$\frac{\partial u}{\partial x} = b e^{\tau t}$

$\frac{\partial^2 u}{\partial x^2} = 0$

Thus, $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2} + \tau u$.

e) $u(t,x) = e^{\lambda t} \sin\left(\left(\frac{\tau - \lambda}{\mu}\right)^{\frac{1}{2}}\right)$.

$\frac{\partial u}{\partial t} = \lambda u(t,x)$

$\frac{\partial u}{\partial x} = \sqrt{\frac{\tau - \lambda}{\mu}} e^{\lambda t} \cos\left(\left(\frac{\tau - \lambda}{\mu}\right)^{\frac{1}{2}}\right)$

$\frac{\partial^2 u}{\partial x^2} = -\frac{\tau - \lambda}{\mu} u(t, x)$

Thus, $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2} + \tau u.$