Math 19. Lecture 15
Introduction to Advection (II)

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1 The Advection Equation

Recall that an *advection equation* has the form

$$\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial t} - ru.$$

The solutions to this equations are of the form

$$u(t, x) = e^{-rt} f(x - ct),$$

where f is any differentiable function in one variable and the choice of f is determined by initial and boundary conditions.

2 Boundary and Initial Conditions

Suppose that we know $u(0, x) = g(x)$ is an initial condition for

$$\frac{\partial u}{\partial t} = -3\frac{\partial u}{\partial t} - ru.$$

That is, the particle density is given by $g(x)$ right before the explosion. Since every solution to this PDE can be written in the form

$$u(t, x) = e^{-rt} f(x - 3t),$$

we know that

$$g(x) = u(0, x) = f(x)$$

or

$$u(t, x) = e^{-rt} g(x - 3t).$$

3 Traveling Wave Solutions

First, observe that $u_t = -3u_x - ru$ predicts the values for $u(t, x)$ at all times $t \geq 0$, and all of the points x. Then $q(t, x) = 3u(t, x)$ is predictive when the value of $u(t, 0)$ is specified for all t. If $u(t, 0) = h(t)$, we say that this is a *boundary condition* for $u_t = -3u_x - ru$. Thus,

$$h(t) = u(t, 0) = e^{-rt} f(-3t),$$

or if we make the substitution $s = -3t$,

$$f(s) = e^{-rs/3} h(-s/3).$$

Therefore, our solution becomes

$$u(t, x) = e^{-rt} e^{-r(x-3t)/3} h((3t - x)/3).$$

In the example of our meltdown, this resembles a *traveling wave*.

Homework

- Chapter 13. Exercises 1, 3, 5, 6, 7, 8; pp. 213–215.

Reading and References

- C. Taubes. *Modeling Differential Equations in Biology*. Prentice Hall, Upper Saddle River, NJ, 2001. Chapter 13.
- "Malaria: Focus on Mosquito Genes" pp. 198–202.