HW #13
Chapter 18: Ex. 1, 2, 3, 5 (a,c), page 309

Ex 1

The equations are:

(7) $\mu\frac{\partial^2 u_e}{\partial x^2} + f(u_e) = 0$
(8) $\frac{\partial u_e}{\partial x}(0) = \frac{\partial u_e}{\partial x}(L) = 0$

$f(u_e) = a \cdot u_e$, $a > 0$
Show that (7) and (8) can be solved with $u_e \neq 0$
if a can be written as $a = \mu\frac{n^2\pi^2}{L^2}$
$\mu\frac{\partial^2 u_e}{\partial x^2} + f(u_e) = 0 \Rightarrow \frac{\partial^2 u_e}{\partial x^2} = -\frac{a}{\mu}u_e = cu_e$
Since $\mu > 0$ and $a > 0$ then $c = \frac{-a}{\mu} < 0$

Thus for $c < 0$, we have:
$u_e(x) = \alpha\cos(\sqrt{\frac{a}{\mu}}x) + \beta\sin(\sqrt{\frac{a}{\mu}}x)$
$u_e'(x) = \sqrt{\frac{a}{\mu}}(-\alpha\sin(\sqrt{\frac{a}{\mu}}x) + \beta\cos(\sqrt{\frac{a}{\mu}}x))$
$u_e'(0) = \beta\sqrt{\frac{a}{\mu}} = 0 \Rightarrow \beta = 0$
$u_e'(L) = -\alpha\sqrt{\frac{a}{\mu}}\sin(\sqrt{\frac{a}{\mu}}L) = 0$
We get two cases:
if $\alpha = 0$, then $u_e(x) = 0$ (which we don't need since we're told to find a solution with $u_e \neq 0$)
otherwise, $\sin(\sqrt{\frac{a}{\mu}}L) = 0 \Rightarrow \sqrt{\frac{a}{\mu}}L = n\pi \Rightarrow \frac{a}{\mu} = \left(\frac{n\pi}{L}\right)^2 \Rightarrow a = \mu\frac{n^2\pi^2}{L^2}$ as given.

Ex. 2

$f(u_e) = -au_e$; $a > 0$. The steps of the algorithm are the same as in the previous exercise.
$\frac{\partial^2 u_e}{\partial x^2} = \frac{a}{\mu}u_e = cu_e \Rightarrow c = \frac{a}{\mu} > 0$ (since $a > 0$, $\mu > 0$)
$u_e(x) = \alpha e^{\sqrt{\frac{a}{\mu}}x} + \beta e^{-\sqrt{\frac{a}{\mu}}x}$
$u_e'(x) = \sqrt{\frac{a}{\mu}}(\alpha e^{\sqrt{\frac{a}{\mu}}x} - \beta e^{-\sqrt{\frac{a}{\mu}}x})$

$u_e'(0) = \sqrt{\frac{a}{\mu}}(\alpha - \beta)$ so $\alpha = \beta$
$u_e'(L) = \sqrt{\frac{a}{\mu}}\alpha(e^{\sqrt{\frac{a}{\mu}}L} - e^{-\sqrt{\frac{a}{\mu}}L}) \Rightarrow \alpha = 0$ and $\beta = 0$
Thus, the only solution is $u_e(x) = 0$

Ex. 3

Achieving this exact ration is highly unlikely. Thus you would not expect tigers to have stripes.

Ex. 5

a) $f(u) = \sin(u)$
   $u_e(x) = \cos(2x)$
   $f(u_e(x)) = \sin(\cos(2x))$
   $f'(u_e(x)) = \cos(\cos(2x))$

c) $f(u) = (1 + u^2)$
   $u_e(x) = 1 + x^2$
   $f(u_e(x)) = 1 + (1 + x^2)^2$
   $f'(u_e(x)) = 2 + 2x^2$