HW #11
Ex. 1, 3 (a,c), 4(a,c,e)

Ex 1.

$u(t,x) = R\frac{1}{\sqrt{t}}e^{-\frac{x^2}{4\mu t}}$.

We have the graph of $u(1,x)$ and of $u(2,x)$. We can find the value of $u(1,x)$ and $u(2,x)$ for any x.

Determine R and μ in each case:

$u(1,x) = Re^{-x^2/4\mu}$

If we take $x = 0$ then $u(1,0) = e^{-0/4\mu} = R$. Therefore, $R = u(1,0)$.

Now, take another value for x.

$u(1,2) = u(1,0)e^{-\frac{4}{4\mu}} \Rightarrow u(1,2) = u(1,0)e^{-\frac{1}{\mu}}$.

$-\frac{1}{\mu} = \ln\left(\frac{u(1,2)}{u(1,0)}\right) \Rightarrow \mu = -\frac{1}{\ln\left(\frac{u(1,2)}{u(1,0)}\right)}$.

For $u(2,x)$, we do the same:

$u(2,0) = \frac{R}{\sqrt{2}}e^{0^2/8\mu} \Rightarrow R = \sqrt{2}u(2,0)$.

$u(2,\sqrt{8}) = \frac{\sqrt{2}u(2,0)}{\sqrt{2}}e^{-8/8\mu} = u(2,0)e^{-\frac{1}{\mu}}$.

Thus, $\mu = -\frac{1}{\ln\left(\frac{u(2,\sqrt{8})}{u(2,0)}\right)}$

Ex. 3

a) See the graphs on the next page.

Figure 1: $\sin(\pi x)$ – positive on $0 \leq x \leq 1$ since the sinus function is positive in the interval $[0, \pi]$

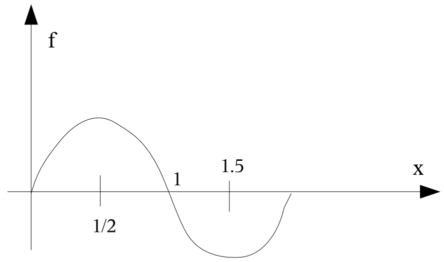


Figure 2: $\sin(3\pi x)$ – positive only in the intervals $0 \leq x \leq 1/3$ and $2/3 \leq x \leq 1$, but negative in between $1/3$ and $2/3$

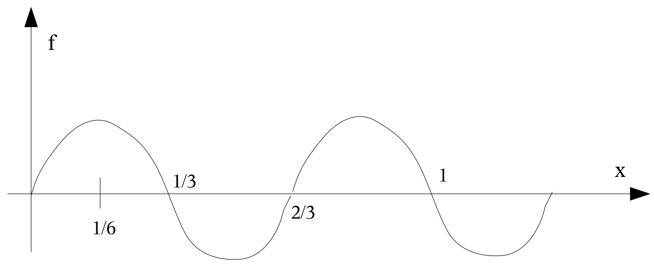
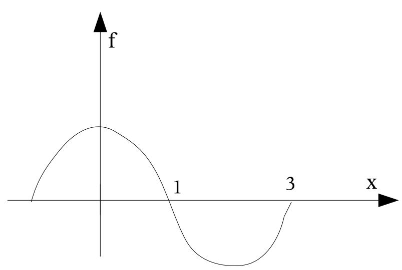


Figure 3: $\cos(\pi x/2)$ – positive in the interval $0 \leq x \leq 1$



c)

Figure 4: $\sin(\pi x/4)$ – positive in the interval $0 \leq x \leq \frac{3}{2}$

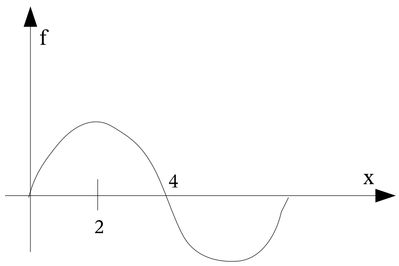


Figure 5: $\sin(\pi x/2)$ – positive in the interval $0 \leq x \leq \frac{3}{2}$

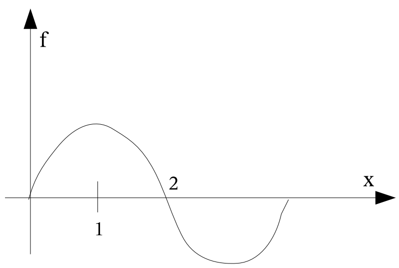


Figure 6: $\sin(3\pi x/4)$ – positive in the interval $0 \leq x \leq 4/3$, but negative in $4/3 < x \leq 3/2$

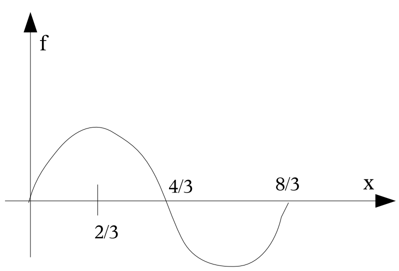
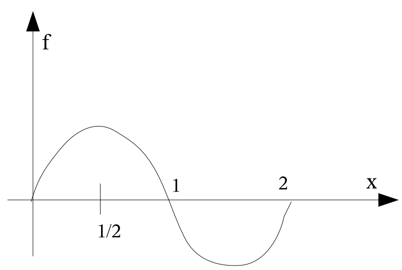


Figure 7: $\sin(\pi x)$ – positive in the interval $0 \le x \le 1$, but negative in the interval $1 < x \le 3/2$



Ex. 4

a) $B(x) = \alpha e^{5x} + \beta e^{-5x}$.

$$\begin{matrix} x = 0 \\ x = 1 \end{matrix} \qquad \begin{cases} \alpha + \beta = 0 \\ \alpha e^5 + \beta e^{-5} = 0 \end{cases}$$

$$\begin{cases} \alpha = -\beta \\ \alpha = -\beta e^{-10} \end{cases}$$

The solution is $\alpha = \beta = 0$.

c) $B(x) = \alpha \cos(\pi x) + \beta \sin(\pi x)$.

$$\begin{matrix} x = 0 \\ x = 1 \end{matrix} \qquad \begin{cases} \alpha \cos 0 + \beta \sin 0 = 0 \\ \alpha \cos \pi + \beta \sin \pi = 0 \end{cases}$$

$$\begin{cases} \alpha = 0 \\ -\alpha = 0 \end{cases}$$

Thus, $\alpha = 0$, $\beta \in \mathbb{R}$

e) $B(x) = \alpha e^{3\pi x} + \beta e^{-3\pi x}$.

$$\begin{matrix} x = 0 \\ x = 1 \end{matrix} \qquad \begin{cases} \alpha + \beta = 0 \\ \alpha e^{3\pi} + \beta e^{-3\pi} = 0 \end{cases}$$

$$\begin{cases} \alpha = -\beta \\ \alpha = -\beta e^{-6\pi} \end{cases}$$

The solution is $\alpha = \beta = 0$.