HW #20 Math 19

Ch 25 # 1, 2, 5, 6 p. 414

Ch 26 # 1 (a, c), 2 (a, c), 3 p. 428

Ch. 25

(1) $\frac{dx}{dt} = 3x + x^2 + 3;$ $x(0) = 0$ $x(t_1) = 1$

Let $\frac{dx}{dt} = f(x)$

$f_{min} \leq \frac{dx}{dt} \leq f_{max} \Rightarrow \int_0^{t_1} f_{min}\, dt \leq \int_0^{t_1} \frac{dx}{dt}\, dt \leq \int_0^{t_1} f_{max}\, dt \Rightarrow$

$\Rightarrow f_{min} \cdot t_1 \leq x(t_1) - x(0) \leq f_{max} \cdot t_1$

Are there max/min. on the interval $0 \leq x \leq 1$?

$\frac{df}{dx} = 3 + 2x = 0 \Rightarrow x = -\frac{3}{2} \notin [0, 1] \Rightarrow$ Thus check endpoints

$f(0) = 3 = f_{min}$

$f(1) = 7 = f_{max}$ $\Rightarrow 3t \leq 1 \leq 7t \Rightarrow \boxed{\frac{1}{7} \leq t \leq \frac{1}{3}}$

(2) $\frac{dx}{dt} = x^3 + 5 = f(x)$ $f(0) = 5 = f_{min};$ $f(1) = 6 = f_{max}$

$f'(x) = 3x^2 = 0 \Rightarrow x = 0$ which is already an endpoint.

Thus $5t_1 \leq 1 \leq 6t_1 \Rightarrow \frac{1}{6} \leq t_1 \leq \frac{1}{5}$

(5) $\frac{dx}{dt} = 2x - x^2 + 1 = f(x)$ $f(0) = 1$ $f(1) = 2$

$f'(x) = 2 - 2x = 0 \Rightarrow x = 1$ which is already an endpoint

Thus $t_1 \leq 1 \leq 2t_1 \Rightarrow \frac{1}{2} \leq t_1 \leq 1$

(6) $\frac{dx}{dt} = 2x + x^2 + 3 = f(x)$ $f(0) = 3$ $f(1) = 6$

$f'(x) = 2 + 2x = 0 \Rightarrow x = -1 \notin [0, 1]$ Thus $f(0) = 3 = f_{min}$ & $f(1) = 6 = f_{max}$

Thus $3t_1 \leq 1 \leq 6t_1 \Rightarrow \frac{1}{6} \leq t_1 \leq \frac{1}{3}$

Ch. 26

① $x, y \in [-10, 10]$

a) $\begin{cases} \frac{dx}{dt} = .1\,x^3 - 3xy \\ \frac{dy}{dt} = .1\,y - .001\,xy \end{cases}$

Try to maximize the values of $\frac{dx}{dt}$ & $\frac{dy}{dt}$ using x, y from the interval given.

You get: $\begin{cases} \frac{dx}{dt} = .1 \cdot 10^3 - 3 \cdot 10 \cdot (-10) = 100 + 300 = 400 \\ \frac{dy}{dt} = .1 \cdot 10 - .001 \cdot (-10)(10) = 1 + 0.1 = 1.1 \end{cases}$

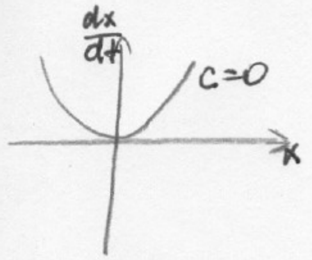
Thus $x(t)$ - fast
$y(t)$ - slow

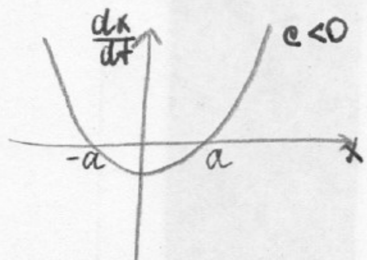
c) $\begin{cases} \frac{dx}{dt} = 0.1x - 0.03xy \\ \frac{dy}{dt} = y + 0.007xy \end{cases}$ => $\begin{cases} \frac{dx}{dt} = .1 \cdot 10 - 0.03 \cdot 10 \cdot (-10) = 1 + 3 = 4 & \text{slow} \\ \frac{dy}{dt} = 10 + 0.007 \cdot 10 \cdot 10 = 10 + .7 = 10.7 & \text{fast} \end{cases}$

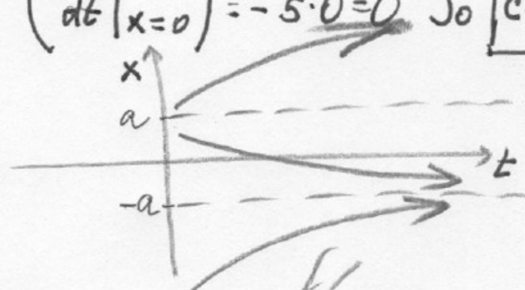
② a) $\frac{dx}{dt} = 5x^2 + c = f(x)$; $f'(x) = 10x = 0$; $c = -\left(\frac{dx}{dt}\Big|_{x=0}\right) = -5 \cdot 0 = 0$ So $\boxed{c=0}$

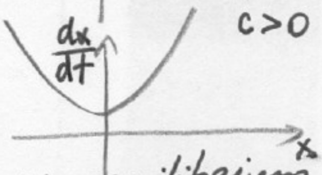
I

II

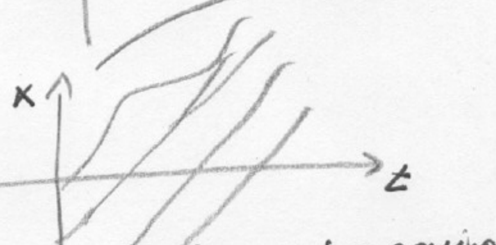








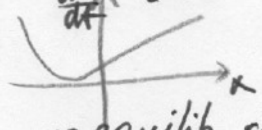
No equilibrium points.



x is always increasing as $t \to \infty$

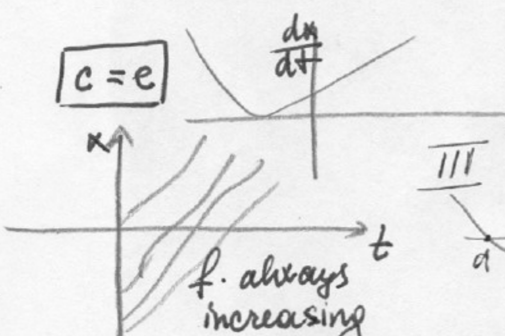
c) I $\frac{dx}{dt} = e^{-x} + cx$

II



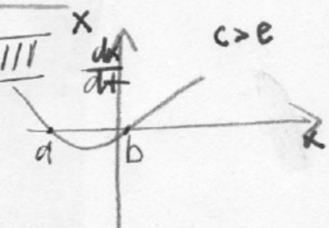
no equilib. sol.

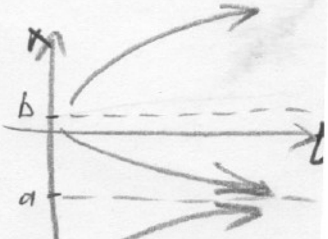
$\boxed{c = e}$



f. always increasing

III





③ Examples of biological systems exhibiting switchlike behavior are systems that are sensitive to a threshold value in their development
- temperature sensitive genes
- action potentials etc.