key

Math 19 Problem Set #4: p. 86 Ex. 1, p. 102-103 Ex. 1, 2, 3, 4ac, 5

Ch. 5 1. There are several correct answers. Here is one:

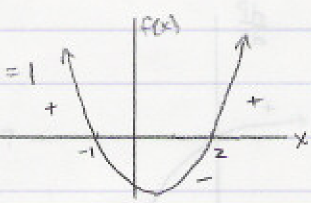
- It is possible to tell if $a > 1$ by observing a population's extinction rate. If the extinction occurs easily, then $a > 1$.
- Now to determine a more specifically, place the same # of R & L snails in a test environment & observe where the population of either R, L or both R & L reaches equilibrium. Count the # of R snails & the # of L snails.
  - If the entire population (i.e. R & L) is at equilibrium, $\frac{dR}{dt} = 0$ & $\frac{dL}{dt} = 0$. Plug the counted # of R & L snails into $\frac{dR}{dt}$ or $\frac{dL}{dt}$ to solve for a.
  - If only the population of R is at equilibrium, you've found an R nullcline. $\frac{dR}{dt} = 0$, so plug obtained #s of R & L snails into $\frac{dR}{dt}$ to find a.
  - If the population of L is constant but R is still changing, you're on an L nullcline. Here $\frac{dL}{dt} = 0$, so plug the # of R & L snails into $\frac{dL}{dt}$ to find a.

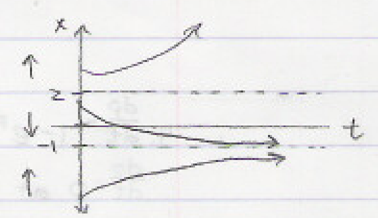
Ch. 6 1. (a) $f(x) = (x+1)(x-2)$, $x(0) = 1$

$f(x) = 0$ at $x = -1, x = 2$

as $t \to \infty$, $x(t) \to -1$

$x(t)$ decreases to equilibrium at $x = -1$



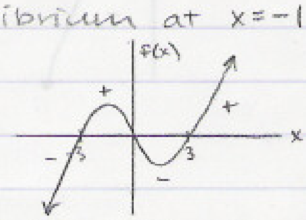


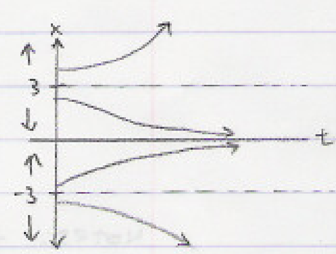
(b) $f(x) = (x+3)x(x-3)$, $x(0) = 1$

$f(x) = 0$ at $x = 0, x = 3, x = -3$

as $t \to \infty$, $x(t) \to 0$

$x(t)$ decreases to equilibrium at $x = 0$



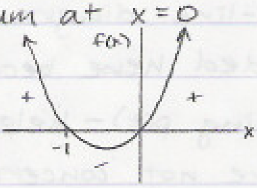


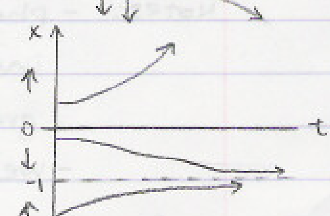
(c) $f(x) = (x+1)x$, $x(0) = 1$

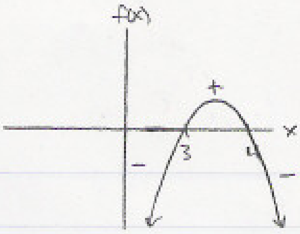
$f(x) = 0$ at $x = 0, x = -1$

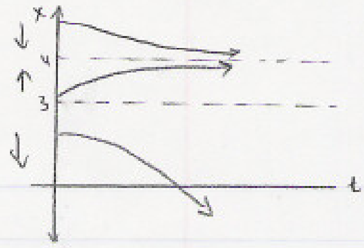
as $t \to \infty$, $x(t) \to \infty$

$x(t)$ increases to ∞.









(d) $f(x) = (3-x)(x-4)$

$f(x) = 0$ at $x = 3, x = 4$

as $t \to \infty$, $x(t) \to -\infty$

$x(t)$ decreases to $-\infty$

2. $x(0) = -4$. as $t \to \infty$, $x(t) \to$?

[see #1 for phase lines]

(a) $x(t)$ increases to equilibrium at $x = -1$

(b) $x(t)$ decreases to $-\infty$

(c) $x(t)$ increases to equilibrium at $x = -1$

(d) $x(t)$ decreases to $-\infty$

3. [see #1 for phase lines]

(a) 2: unstable — -1: stable

(b) -3: unstable — 0: stable — 3: stable

(c) -1: stable — 0: unstable

(d) 3: unstable — 4: stable

4. first-order Taylor expansion for $f(x)$ at $x_0 = 0$ is $g(x) = f(0) + f'(0)x$

(a) $f(x) = xe^{-6x}$

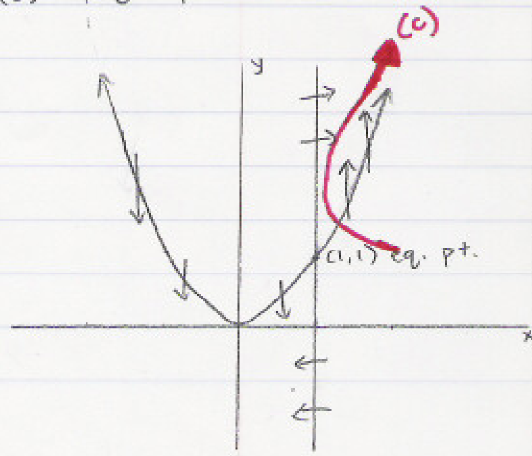
$f(0) = 0$ $\quad f'(x) = e^{-6x} - 6xe^{-6x}$ $\quad f'(0) = 1 - 0 = 1$

$g(x) = 0 + 1(x) = x$

(c) $f(x) = 4x + x^2$

$f(0) = 0$ $\quad f'(x) = 4 + 2x$ $\quad f'(0) = 4 + 0 = 4$

$g(x) = 0 + 4x = 4x$



5. $\frac{dx}{dt} = y - x^2$, $\frac{dy}{dt} = x - 1$

(a) x null clines: $\frac{dx}{dt} = 0 = y - x^2$ $\quad y = x^2$

y null clines: $\frac{dy}{dt} = 0 = x - 1$ $\quad x = 1$

(b) x: $\frac{dy}{dt}(2,4) = 2 - 1 = 1$ ↑, $\frac{dy}{dt}(0,0) = 0 - 1 = -1$ ↓

y: $\frac{dx}{dt}(1,0) = -1$ ←, $\frac{dx}{dt}(1,2) = 1$ →

(d) $x(t) < x(0)$

$y(t) < y(0)$