Problem Set #7
Chapter 10, Ex. 1,3,4, p.172;
Chapter 11: Ex. 1,2,3,4, p.177

November 2, 2004

Ex. 1, p. 172

$$\frac{d}{dt}\binom{x}{y} = \binom{x - x^2 - 2xy}{2y - y^2 - 3xy}$$

Equilibrium points are: $\binom{0}{0}, \binom{1}{0}, \binom{0}{2}, \binom{0.6}{0.2}$.

$$\mathcal{D} = \begin{pmatrix} 1 - 2x - 2y & -2x \\ -3y & 2 - 2y - 3x \end{pmatrix}$$

Equilib.point	Matrix	$Det(\mathcal{D})$	$Tr(\mathcal{D})$	Stability
$\binom{0}{0}$	$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$	2 $(2 > 0)$	3 $(3 > 0)$	Unstable
$\binom{0}{2}$	$\begin{pmatrix} -3 & 0 \\ -6 & -2 \end{pmatrix}$	6 $(6 > 0)$	-5 $(-5 < 0)$	Stable
$\binom{1}{0}$	$\begin{pmatrix} -1 & -2 \\ 0 & -1 \end{pmatrix}$	1 $(1 > 0)$	-2 $(-2 < 0)$	Stable
$\binom{0.6}{0.2}$	$\begin{pmatrix} -0.6 & -1.2 \\ -0.6 & -0.2 \end{pmatrix}$	-0.6 $(-0.6 < 0)$	-0.8 $(-0.8 < 0)$	Unstable

Ex. 3, p.172

- $h = x^2y^3$
  $\frac{\partial h}{\partial x} = 2y^3x$
  $\frac{\partial h}{\partial y} = 3x^2y^2$
  $\frac{\partial^2 h}{\partial x^2} = 2y^3$
  $\frac{\partial^2 h}{\partial y^2} = 6x^2y$
  $\frac{\partial^2 h}{\partial y \partial x} = 6xy^2$

$\frac{\partial^2 h}{\partial x \partial y} = 6xy^2$
The last two are equal.

- $h = x\cos(xy)$
  $\frac{\partial h}{\partial x} = -xy\sin(xy) + \cos(xy)$
  $\frac{\partial h}{\partial y} = -x^2\sin(xy)$
  $\frac{\partial^2 h}{\partial x^2} = -xy^2\cos(xy) - 2y\sin(xy)$
  $\frac{\partial^2 h}{\partial y^2} = -x^3\cos(xy)$
  $\frac{\partial^2 h}{\partial y \partial x} = -2x\sin(xy) - x^2y\cos(xy)$
  $\frac{\partial^2 h}{\partial x \partial y} = -2x\sin(xy) - x^2y\cos(xy)$
  The last two are equal.

- $h = \sin(x + y^2)$
  $\frac{\partial h}{\partial x} = \cos(x + y^2)$
  $\frac{\partial h}{\partial y} = 2y\cos(x + y^2)$
  $\frac{\partial^2 h}{\partial x^2} = -\sin(x + y^2)$
  $\frac{\partial^2 h}{\partial y^2} = 2cos(x + y^2) - 4y^2\sin(x + y^2)$
  $\frac{\partial^2 h}{\partial y \partial x} = -2y\sin(x + y^2)$
  $\frac{\partial^2 h}{\partial x \partial y} = -2y\sin(x + y^2)$
  The last two are equal.

- $h = xe^y$
  $\frac{\partial h}{\partial x} = e^y$
  $\frac{\partial h}{\partial y} = xe^y$
  $\frac{\partial^2 h}{\partial x^2} = 0$
  $\frac{\partial^2 h}{\partial y^2} = xe^y$
  $\frac{\partial^2 h}{\partial y \partial x} = e^y$
  $\frac{\partial^2 h}{\partial x \partial y} = e^y$
  The last two are equal.

Thus, we have seen through these examples that $\frac{\partial^2 h}{\partial y \partial x} = \frac{\partial^2 h}{\partial x \partial y}$.

Ex. 4, p. 172

Integrate $h(x,y)$ over the indicated rectangle.

a)

$$\int_{-1}^{2}\int_{0}^{1} dxdy = \int_{-1}^{2} x|_0^1 \; dy = y|_{-1}^{2} = 3$$

b)

$$\int_{0}^{1}\int_{-1}^{1} x \; dxdy = \int_{0}^{1} (x^2/2)|_{-1}^{1} \; dy = 0$$

c)

$$\int_{0}^{1}\int_{0}^{1} (x+y) \; dxdy = \int_{0}^{1} (x^2/2+yx)|_0^1 \, dy = \int_{0}^{1} (1/2+y) \, dy = (y/2+y^2/2)|_0^1 = 1/2+1/2 = 1$$

d)

$$\int_{2}^{3}\int_{-2}^{1} xy \; dxdy = \int_{2}^{3} (x^2y/2)|_{-2}^{1} \, dy = \int_{2}^{3} \frac{y}{2}(1-4) \, dy = \int_{2}^{3} -\frac{3}{2}y \, dy = (-3y^2/4)|_2^3 = -15/4$$

e)

$$\int_{0}^{1}\int_{0}^{1} \cos(xy) \; dxdy = \int_{0}^{1} \left.\frac{\sin(xy)}{y}\right|_0^1 \; dy = \int_{0}^{1} \frac{\sin y}{y} \; dy = 0.946$$

Chapter 11. Ex. 1, p.177

a) $M = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$\vec{v} = \binom{1}{1}$

$M \; \vec{v} = \binom{1\cdot 1+2\cdot 1}{0\cdot 1+1\cdot 1} = \binom{3}{1}$

b) $M = \begin{pmatrix} 3 & 2 \\ -5 & 4 \end{pmatrix}$

$\vec{v} = \binom{1}{3}$

$M \; \vec{v} = \binom{9}{7}$

c) $M = \begin{pmatrix} 0.5 & 0.2 \\ 8 & 0.4 \end{pmatrix}$

$\vec{v} = \binom{0}{1}$

$M \; \vec{v} = \binom{0.2}{0.4}$

Ex. 2, p.177

	a	b	c
MM'	$\begin{pmatrix} 4 & 17 \\ 1 & 6 \end{pmatrix}$	$\begin{pmatrix} 9 & 9 \\ 2 & 11 \end{pmatrix}$	$\begin{pmatrix} 15 & -2 \\ 18 & -15 \end{pmatrix}$
MM'	$\begin{pmatrix} 2 & 9 \\ 1 & 8 \end{pmatrix}$	$\begin{pmatrix} 15 & -2 \\ 3 & 5 \end{pmatrix}$	$\begin{pmatrix} 9 & 54 \\ 2 & -9 \end{pmatrix}$

Ex. 3, p. 177

	a	b	c
$\det(M)$	1	-27	7
$\det(M)$	2	-4	8

Ex. 4, p. 177

Verify that $\vec{v} = \binom{1}{0}$ is an eigenvector for $M = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

$M\vec{v} = \lambda\vec{v}$

$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\binom{x}{y} = \lambda\binom{x}{y}$

$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\binom{x}{y} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\binom{x}{y} = 0$

$\begin{pmatrix} 1-\lambda & 2 \\ 0 & 1-\lambda \end{pmatrix}\binom{x}{y} = \vec{0}$

$\begin{cases} (1-\lambda)x + 2y = 0 \\ (1-\lambda)y = 0 \end{cases}$

$\binom{x}{y} \neq \vec{0}$

Thus, $1 - \lambda = 0 \;\Rightarrow\; \lambda = 1$. The eigenvalue is $\lambda = 1$. Verify that $\binom{1}{0}$ is an eigenvector:

$M\binom{1}{0} = \lambda\binom{1}{0}$

$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\binom{1}{0} = \lambda\binom{1}{0}$

Thus, $\binom{1}{0}$ is an eigenvector for M.